$W = LS(\{\sin x, \cos x\})$ over $\mathbb{R}$ — ⊛

let $\theta \in \mathbb{R} \Rightarrow \sin\theta, \cos\theta \in \mathbb{R}$

$f_1 = \sin(x+\theta) = (\cos\theta)\cdot\sin x + (\sin\theta)\cdot\cos x$

$= k_1 \sin x + k_2 \cos x \quad \in W$

$f_2 = \cos(x-\theta) = (\cos\theta)\cdot\cos x + (\sin\theta)\cdot\sin x$

$= k_1 \cos x + k_2 \sin x \quad \in W$

Let $a, b \in \mathbb{R}$ s.t.

$0 = af_1 + bf_2 = a[k_1 \sin x + k_2 \cos x] + b[k_1 \cos x + k_2 \sin x]$

$= (ak_1 + bk_2)\sin x + (ak_2 + bk_1)\cos x$

$\because \{\sin x, \cos x\}$ are L.I.

$\Rightarrow \quad ak_1 + bk_2 = 0$ — ① where $k_1 = \cos\theta$ $(\theta \in \mathbb{R})$

$ak_2 + bk_1 = 0$ — ② $\quad k_2 = \sin\theta$

① $\times k_2$ − ② $\times k_1$,

$ak_1k_2 + bk_2^2 = 0$

$ak_1k_2 + bk_1^2 = 0$

$b(k_2^2 - k_1^2) = 0 \Rightarrow b = 0$

b'coz $k_1^2 = k_2^2$ not possible $\forall\, \theta \in \mathbb{R}$

hence by ①, $a = 0$ as $k_1 \neq 0 \ \forall\, \theta \in \mathbb{R}$

$\therefore 0 = af_1 + bf_2 \Rightarrow a, b = 0$

$\Rightarrow S = \{f_1, f_2\}$ are L.I.

by (A) $\dim(W) = 2$

and $|S'| = 2 \Rightarrow W = LS(S')$

$\therefore$ S' form basis of W

(a, b, d)

A

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