

$$W = \text{LS}(\{\sin x, \cos x\}) \text{ over } \mathbb{R} \quad \text{--- } \textcircled{1}$$

$$\text{Let } \theta \in \mathbb{R} \Rightarrow \sin \theta, \cos \theta \in \mathbb{R}$$

$$\begin{aligned} f_1 &= \sin(x+\theta) = (\cos \theta) \cdot \sin x + (\sin \theta) \cdot \cos x \\ &= k_1 \sin x + k_2 \cos x \in W \end{aligned}$$

$$\begin{aligned} f_2 &= \cos(x-\theta) = (\cos \theta) \cdot \cos x + (\sin \theta) \cdot \sin x \\ &= k_1 \cos x + k_2 \sin x \in W \end{aligned}$$

Let  $a, b \in \mathbb{R}$  s.t.

$$\begin{aligned} 0 &= af_1 + bf_2 = a[k_1 \sin x + k_2 \cos x] + b[k_1 \cos x + k_2 \sin x] \\ &= (ak_1 + bk_2) \sin x + (ak_2 + bk_1) \cos x \end{aligned}$$

$\therefore \{\sin x, \cos x\}$  are L.I.

$$\Rightarrow \begin{aligned} ak_1 + bk_2 &= 0 \quad \textcircled{1} \quad \text{where } k_1 = \cos \theta \quad (\theta \in \mathbb{R}) \\ ak_2 + bk_1 &= 0 \quad \textcircled{2} \quad k_2 = \sin \theta \end{aligned}$$

$$\textcircled{1} \times k_2 - \textcircled{2} \times k_1, \quad \begin{aligned} ak_1k_2 + bk_2^2 &= 0 \\ ak_1k_2 + bk_1^2 &= 0 \end{aligned}$$

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$$b(k_2^2 - k_1^2) = 0 \Rightarrow b = 0$$

b'coz  $k_1^2 = k_2^2$  not possible  $\forall \theta \in \mathbb{R}$

hence by  $\textcircled{1}$ ,  $a = 0$  as  $k_1 \neq 0 \forall \theta \in \mathbb{R}$

$$\begin{aligned} \therefore 0 &= af_1 + bf_2 \Rightarrow a, b = 0 \\ &\Rightarrow S' = \{f_1, f_2\} \text{ are L.I.} \end{aligned}$$

by ④  $\dim(W) = 2$

and  $|S'| = 2 \Rightarrow W = \text{LS}(S')$

$\therefore S'$  form basis of  $W$

$(a, b, d)$

A

AE

PA

CE