$\Rightarrow$ Molar heat capacity at constant pressure is given by

$$C_p = \left(\frac{\partial H}{\partial T}\right)_P$$

and that at constant volume by the relation

$$C_v = \left(\frac{\partial U}{\partial T}\right)_V$$

The difference between C_p and C_v is

$$C_p - C_v = \left(\frac{\partial H}{\partial T}\right)_P - \left(\frac{\partial U}{\partial T}\right)_V \quad \text{-------- (1)}$$

Again, $H = U + PV$, where H is enthalpy, U is internal energy, P = pressure and V is the volume. Hence equation (1) becomes

$$C_p - C_v = \left[\frac{\partial (U + PV)}{\partial T}\right]_P - \left(\frac{\partial U}{\partial T}\right)_V$$

$$= \left(\frac{\partial U}{\partial T}\right)_P + P\left(\frac{\partial V}{\partial T}\right)_P - \left(\frac{\partial U}{\partial T}\right)_V \quad \text{-------- (2)}$$

since $U = f(V, T)$; T = temperature.

$$\therefore \quad dU = \left(\frac{\partial U}{\partial V}\right)_T dV + \left(\frac{\partial U}{\partial T}\right)_V dT$$

Dividing by dT at constant pressure, we get

$$\left(\frac{\partial U}{\partial T}\right)_P = \left(\frac{\partial U}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P + \left(\frac{\partial U}{\partial T}\right)_V$$

substituting the value of $\left(\frac{\partial U}{\partial T}\right)_P$ in equation-②, we get

$$c_p - c_v = \left(\frac{\partial U}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P + P\left(\frac{\partial V}{\partial T}\right)_P$$

$$\therefore \quad \boxed{c_p - c_v = \left[P + \left(\frac{\partial U}{\partial V}\right)_T\right]\left(\frac{\partial V}{\partial T}\right)_P} \quad \text{------} \quad ③$$

This is the general relation between c_p and c_v. This equation is applicable for any materials i.e., solid, liquid and gas (ideal and real).