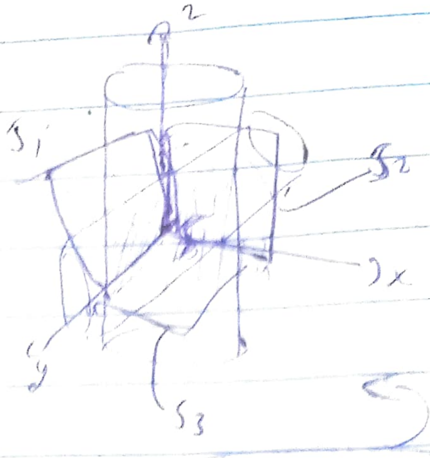
Since surface is included in first octant.
So, bound it by xy, yz, zx planes and make it a closed region



Now, we can apply divergence theorem

$\boxed{S' = S \cup S_1 \cup S_2 \cup S_3}$ total bounded surface

$S_3 \Rightarrow$ xy plane
$S_2 \Rightarrow$ xz plane
$S_1 \rightarrow$ yz plane
$S \rightarrow$ intersection of $x^2+y^2=a^2$ and $x^2+z^2=a^2$

$$\iint_{S'} \nabla\cdot(\nabla\times A)\hat{n}\,dS = \iint_{S_1}(\nabla\times A)\hat{n}\,dS + \iint_{S_2}(\nabla\times A)\hat{n}\,dS + \iint_{S_3}(\nabla\times A)\hat{n}\,dS + \iint_{S}(\nabla\times A)\hat{n}\,dS$$

$$0 = \text{''}$$

$$\Rightarrow \iint_S (\nabla\times A)\hat{n}\,dS = -\left(\iint_{S_1}(\nabla\times A)\hat{n}\,dS_1 + \iint_{S_2}(\nabla\times A)\hat{n}\,dS_2 + \iint_{S_3}(\nabla\times A)\hat{n}\,dS\right) \quad (1)$$

(1) for S_1 = yz plane

$\hat{n} = -\hat{i}$, $x=0$

$$\iint_{S_1}(\nabla\times A)(-\hat{i})\,dS_1 = \iint 0 = \boxed{0}$$

$$\nabla\times A = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2yz & -(x+3y-2) & x^2+z \end{vmatrix}$$

$$\Rightarrow \hat{i}(0-0) - \hat{j}(2x-2y) + \hat{k}(-1-2z)$$

$$= \boxed{(2y-2x)\hat{j} + \hat{k}(-1-2z)}$$

(2) for S_2 xz plane

$\hat{n} = -\hat{j}$

$$\iint_{S_2}(\nabla\times A)(-\hat{j})\,dS = -\iint(2y-2x)\,dS$$

$$= 2\iint x\,dx\,dz$$

$$2\int_{x=0}^{a}\int_{z=0}^{\sqrt{a^2-x^2}} x\,dz\,dx = 2\int_{x=0}^{a} x\sqrt{a^2-x^2} = \left[\frac{(a^2-x^2)^{3/2}}{3/2}\right]_0^a$$

$$= \boxed{\frac{2}{3}a^3}$$

(3) for S_3 = xy plane

$\hat{n} = -\hat{k}$

$$\iint_{S_3} (\nabla\times A)\cdot\hat{k}\,dS = -\iint(-1-2z)\,dx\,dy = -\iint(-1)\,dx\,dy = \iint dx\,dy$$

$$= \boxed{\frac{\pi a^2}{4}}$$

from (1)

$$\iint_S (\nabla\times A)\cdot\hat{n}\,dS = -0 - \frac{\pi a^2}{4} - \frac{2}{3}a^3$$

$$= \frac{-3\pi a^2 - 8a^3}{12} \Rightarrow \boxed{-\frac{a^2}{12}(3\pi+8a)}$$