

If T is linear Transformation from $\mathbb{R}^2 \rightarrow \mathbb{R}^2$
for reflection across line $y = mx$
given by

$$T(a, b) = \left(2 \frac{(a + mb)}{1 + m^2} - a, 2 \left(\frac{m(a + mb)}{1 + m^2} \right) - b \right)$$

then just find $T(e_1)$ & $T(e_2)$
where $e_1 = (1, 0)$ & $e_2 = (0, 1)$

then you will get your answer.

$$T(a,b) = \left(2 \left(\frac{a+mb}{1+m^2} \right) - a, 2 \left(\frac{m(a+mb)}{1+m^2} \right) - b \right)$$

$$T(1,0) = \left(2 \left(\frac{1+m \cdot 0}{1+m^2} \right) - 1, 2 \left(\frac{m(1+m \cdot 0)}{1+m^2} \right) - 0 \right)$$

$$= \left(\frac{2}{1+m^2} - 1, \frac{2m}{1+m^2} \right)$$

$$= \left(\frac{2 - (1+m^2)}{1+m^2}, \frac{2m}{1+m^2} \right) = \boxed{\left(\frac{1-m^2}{1+m^2}, \frac{2m}{1+m^2} \right)}$$

$$T(0,1) = \left(2 \left(\frac{0+m \cdot 1}{1+m^2} \right) - 0, 2 \left(\frac{m(0+m \cdot 1)}{1+m^2} \right) - 1 \right)$$

$$= \left(\frac{2m}{1+m^2}, \frac{2m^2}{1+m^2} - 1 \right) \Rightarrow \left(\frac{2m}{1+m^2}, \frac{2m^2 - (1+m^2)}{1+m^2} \right)$$

$$\left(2m, m^2 - 1 \right)$$

$$= \left(\frac{2}{1+m^2} - 1, \frac{2m}{1+m^2} \right)$$

$$= \left(\frac{2 - (1+m^2)}{1+m^2}, \frac{2m}{1+m^2} \right) = \boxed{\left(\frac{1-m^2}{1+m^2}, \frac{2m}{1+m^2} \right)}$$

$$T(0,1) = \left(2 \left(\frac{0+m \cdot 1}{1+m^2} \right) - 0, 2 \left(\frac{m(0+m \cdot 1)}{1+m^2} - 1 \right) \right)$$

$$= \left(\frac{2m}{1+m^2}, \frac{2m^2}{1+m^2} - 1 \right) \Rightarrow \left(\frac{2m}{1+m^2}, \frac{2m^2 - (1+m^2)}{1+m^2} \right)$$

$$= \boxed{\left(\frac{2m}{1+m^2}, \frac{m^2-1}{1+m^2} \right)}$$

$$\text{Then } [T]_{\beta} = \begin{bmatrix} \frac{1-m^2}{1+m^2} & \frac{2m}{1+m^2} \\ \frac{2m}{1+m^2} & \frac{m^2-1}{1+m^2} \end{bmatrix} = \frac{1}{m^2+1} \begin{bmatrix} 1-m^2 & 2m \\ 2m & m^2-1 \end{bmatrix}$$