1. y=f(t), thus y''=f''(t) and y'=f'(t)

2. y(0)=f(t=0)=0 and y'(0)=f'(t=0)=1

3. Rewriting the eqn. **f''(t)+t f'(t)-2f(t)=6-t**

4. Laplace transforms=> $L(f(t)) = f(s)$ and Inverse Laplace transforms=> $L^{-1}(f(s)) = f(t)$

3=> **f''(t)+t f'(t)-2f(t)=6-t**

Taking laplace transorms on both sides we get

$$s^2 f(s) - sf(t=0) - f'(t=0)$$
$$+ (-1)\frac{d}{ds}(sf(s) - f(t=0)) - 2f(s) = \frac{6}{s} - (s^{-2})$$

substituting values given from 2

$$s^2 f(s) - 0 - 1 - \frac{d}{ds}(sf(s) - 0) - 2f(s) = \frac{6}{s} - \frac{1}{s^2}$$

expanding the derivative

$$s^2 f(s) - 1 - f(s) - sf'(s) - 2f(s) = \frac{6}{s} - \frac{1}{s^2}$$

rearrenging we have

$$(s^2-3)f(s) - sf'(s) = \frac{6}{s} - \frac{1}{s^2} - 1$$

=> $f'(s) + (\frac{3}{s} - s)f(s) = \frac{1}{s^3} - \frac{6}{s} - \frac{1}{s}$

{the above equation looks like $\frac{dy}{dx}+Py=Q$

so lets take a detour into diferential equations}

IF= $e^{\int(\frac{3}{s}-s))ds}$

=>IF=$e^{3ln(s)-(\frac{s^2}{2})}$

=>IF= $\frac{s^3}{e^{\frac{s^2}{2}}}$

so the solution of the differntial equation is

$f(s)*IF=\int[\frac{1}{s^3}-\frac{6}{s^2}-\frac{1}{s}](IF)ds$

=> $f(s)(\frac{s^3}{e^{\frac{s^2}{2}}})=\int[\frac{1}{s^3}-\frac{6}{s^2}-\frac{1}{s}][\frac{s^3}{e^{\frac{s^2}{2}}}]ds$

=> $f(s)(\frac{s^3}{e^{\frac{s^2}{2}}})=\int[\frac{1}{e^{\frac{s^2}{2}}}-\frac{6s}{e^{\frac{s^2}{2}}}-\frac{s^2}{e^{\frac{s^2}{2}}}]ds$

for RHS we can use integrating by parts

{1. $\int\frac{1}{e^{\frac{s^2}{2}}}ds=\frac{s}{e^{\frac{s^2}{2}}}$

2. $\int\frac{6s}{e^{\frac{s^2}{2}}}ds=\frac{-6(s+1)}{e^{s^2/2}}$

}

=> $f(s)(\frac{s^3}{e^{\frac{s^2}{2}}}) = \frac{s}{e^{\frac{s^2}{2}}} + \frac{6(s+1)}{e^{\frac{s^2}{2}}} + \frac{2s-2}{e^{\frac{s^2}{2}}}$

cancelling common terms, i.e, $\frac{1}{e^{\frac{s^2}{2}}}$, we get

$f(s)((s^3) = \frac{9}{s^2} + (4)$

=> $f(s) = \frac{9}{s^2} + \frac{4}{s^3}$

taking inverse laplace tranform

$f(t) = 9(t) + 4\frac{t^2}{2}$

=> $f(t) = 9(t) + 2(t^2)$

Thus solution of $y'' + ty' - 2y = 6 - t$ is
$y = 9(t) + 2(t^2)$