Proof . $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$

$$\nabla^2(f(r)) = \nabla \cdot \nabla f(r)$$

$$\nabla f(r) = \sum \hat{i} \frac{\partial}{\partial x} f(r) = \sum \hat{i} f'(r) \frac{\partial r}{\partial x}$$

$$= \sum \hat{i} f'(r) \frac{x}{r}$$

$$= \frac{f'(r)}{r} \sum \hat{i} x = \frac{f'(r)}{r}(x\hat{i} + y\hat{j} + z\hat{k})$$

$$= \boxed{\frac{f'(r)}{r} \vec{r}}$$

Now

$$\nabla^2 f(r) = \nabla \cdot \left( \frac{f'(r)}{r} \vec{r} \right)$$

$$= \sum \hat{i} \frac{\partial}{\partial x} \left( \frac{1}{r} f'(r) \vec{r} \right)$$

$$= \sum \hat{i} \left[ \frac{\partial}{\partial x}\left(\frac{1}{r}\right) f'(r) \vec{r} + \frac{1}{r} \frac{\partial}{\partial x}(f'(r)) \vec{r} + \frac{1}{r} f'(r) \frac{\partial \vec{r}}{\partial x} \right]$$

$$= \sum \hat{i} \cdot \left[ -\frac{1}{r^2} \cdot \frac{x}{r} f'(r) \vec{r} + \frac{1}{r} f''(r) \frac{\partial r}{\partial x} \vec{r} + \frac{1}{r} f'(r) \hat{i} \right]$$

$$= \sum \left[ -\frac{f'(r)}{r^3} \cdot x(\hat{i} \cdot \vec{r}) + \frac{1}{r} f''(r) \frac{x}{r} \cdot (\hat{i} \cdot \vec{r}) + \frac{1}{r} f'(r) \hat{i} \cdot \hat{i} \right]$$

$$= -\frac{f'(r)}{r^3} \sum x^2 + \frac{f''(r)}{r^2} \sum x^2 + \frac{1}{r} f'(r) \sum 1$$

$$= -\frac{f'(r)}{r} + f''(r) + \frac{3 f'(r)}{r} \qquad \left[ \sum 1 = 3 \right]$$

$$= \boxed{f''(r) + \frac{2}{r} f'(r)}$$