Prove that $\quad \nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$

$$\nabla^2 f(r) \Rightarrow \nabla \cdot (\nabla f(r)) \Rightarrow \nabla \cdot \left(f'(r) \frac{\vec{r}}{r}\right)$$

$$\Rightarrow \nabla \cdot \left(\underbrace{\frac{f'(r)}{r}}_{\text{scalar}} \underbrace{\vec{r}}_{\text{vector}}\right)$$

$$\Rightarrow \nabla \frac{f'(r)}{r} \cdot \vec{r} + \frac{f'(r)}{r} \nabla \cdot \vec{r}$$

$$\Rightarrow \left(\frac{r f''(r) - f'(r)}{r^2}\right) \frac{\vec{r}}{r} \cdot \vec{r} + \frac{3 f'(r)}{r}$$

$$\Rightarrow \left(\frac{r f''(r) - f'(r)}{r}\right) + \frac{3 f'(r)}{r}$$

$$\Rightarrow f''(r) - \frac{1}{r} f'(r) + \frac{3}{r} f'(r)$$

$$\Rightarrow f''(r) + \frac{2}{r} f'(r)$$

$$f''(r) + \frac{2}{r} f'(r) = 0 \qquad \text{[from above]}$$

$$\int \frac{f''(r)}{f'(r)} = \int -\frac{2}{r}$$

$$\log f'(n) = -2\log n + \log C_1$$

$$f'(n) = \frac{C}{n^2}$$

$$f(n) = -\frac{C}{n} + C_2 .$$

$$\Rightarrow f(n) = \frac{C_1}{n} + C_2$$ → harmonic fn.

C_1, C_2 are constant