

Q. Relationship between AR, MR and Elasticity of Demand

$$E = \frac{A}{A - M}$$

This equation actually shows the relationship between AR, MR and Elasticity of demand where

E = Elasticity of Demand

A = AR (Average Revenue) and

M = MR (Marginal Revenue).

Now, we will try ~~to~~ derive this relationship:

Let $Q = f(P)$ be the demand function.

Then,

Elasticity of Demand,

$$E = \frac{dQ}{dP} \cdot \frac{P}{Q} \quad \text{--- (i)}$$

Since, TR (Total Revenue) $= P \times Q$

$$\Rightarrow MR = \frac{d}{dQ} (TR)$$

$$\Rightarrow MR = \frac{d}{dQ} (PQ)$$

$$\Rightarrow MR = P \frac{dQ}{dQ} + Q \frac{dP}{dQ}$$

$$\Rightarrow MR = P + Q \frac{dP}{dQ}$$

$$\Rightarrow MR = P \left(1 + \frac{Q}{P} \cdot \frac{dP}{dQ} \right)$$

$$\therefore E = \frac{dQ}{dP} \cdot \frac{P}{Q}$$

$$\Rightarrow \frac{1}{E} = \frac{dP}{dQ} \cdot \frac{Q}{P}$$

$$\therefore MR = P \left(1 + \frac{1}{E} \right)$$

$$\therefore P = AR$$

$$\therefore MR = AR \left(1 + \frac{1}{E} \right)$$

$$\Rightarrow \cancel{MR} \cdot AR \cdot \frac{1}{E} = AR - MR$$

$$\Rightarrow \frac{1}{E} = \frac{AR - MR}{AR}$$

$$\Rightarrow E = \frac{AR}{AR - MR}$$

$$\Rightarrow \boxed{E = \frac{A}{A - M}}$$