$$\frac{\partial(\vec{r}\cdot\vec{r})}{\partial t} = \vec{r}\cdot\frac{\partial\vec{r}}{\partial t} + \vec{r}\cdot\frac{\partial\vec{r}}{\partial t}$$

$$= 2\,\vec{r}\cdot\frac{\partial\vec{r}}{\partial t}$$

Consider

$$\int_2^3 \left(r\cdot\frac{\partial r}{\partial t}\right)\partial t = \frac{1}{2}\int_2^3 \frac{\partial(\vec{r}\cdot\vec{r})}{\partial t}\,\partial t$$

$$\Rightarrow \frac{1}{2}\int_2^3 \partial(r^2)$$

$$= \frac{1}{2}\left[r^2\right]_2^3$$

$$\Rightarrow \frac{1}{2}\left[(r(3))^2 - (r(2))^2\right]$$

$$\Rightarrow \frac{1}{2}\left[(4\hat{i}+2\hat{j}+3\hat{k})^2 - (2\hat{i}+\hat{j}+2\hat{k})^2\right]$$

$$\Rightarrow \frac{1}{2}\left[16+4+9-(4+1+4)\right]$$

$$\Rightarrow \frac{1}{2}[20] = 10\,J$$