Problem:

$$\int \frac{1}{\sqrt{\sin^3(x)\sin(x+a)}}\,\mathrm{d}x$$

Rewrite/simplify:

$$= \int \frac{1}{\sin^{\frac{3}{2}}(x)\sqrt{\sin(x+a)}}\,\mathrm{d}x$$

Apply addition formulas and/or multiple-angle/argument formulas:

$$\sin(\mathrm{x} \pm \mathrm{y}) = \sin(\mathrm{x})\cos(\mathrm{y}) \pm \cos(\mathrm{x})\sin(\mathrm{y}),$$
$$\cos(\mathrm{x} \pm \mathrm{y}) = \cos(\mathrm{x})\cos(\mathrm{y}) \mp \sin(\mathrm{x})\sin(\mathrm{y})$$

$$= \int \frac{1}{\sin^{\frac{3}{2}}(x)\sqrt{\cos(a)\sin(x)+\sin(a)\cos(x)}}\,\mathrm{d}x$$

Prepare for substitution, use:

$$\sin(x) = \frac{1}{\csc(x)},$$
$$\cos(x) = \frac{\cot(x)}{\csc(x)}$$

$$= \int -\left(-\frac{1}{\sqrt{\sin(a)\cot(x)+\cos(a)}}\right)\csc^2(x)\,\mathrm{d}x$$

Substitute $u = \cot(x) \longrightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = -\csc^2(x)$ (steps)

$$\longrightarrow \mathrm{d}x = -\frac{1}{\csc^2(x)}\,\mathrm{d}u:$$

$$= -\int \frac{1}{\sqrt{\sin(a)u+\cos(a)}}\,\mathrm{d}u$$

... or choose an alternative:

Substitute $\tan(x)$

Now solving:

$$\int \frac{1}{\sqrt{\sin(a)u + \cos(a)}}\, \mathrm{d}u$$

Substitute $v = \sin(a)u + \cos(a) \longrightarrow \dfrac{\mathrm{d}v}{\mathrm{d}u} = \sin(a)$

(steps) $\longrightarrow \mathrm{d}u = \dfrac{1}{\sin(a)}\, \mathrm{d}v$:

$$= \frac{1}{\sin(a)} \int \frac{1}{\sqrt{v}}\, \mathrm{d}v$$

Now solving:

$$\int \frac{1}{\sqrt{v}}\, \mathrm{d}v$$

Apply power rule:

$$\int v^{\mathrm{n}}\, \mathrm{d}v = \frac{v^{\mathrm{n}+1}}{\mathrm{n}+1} \text{ with } \mathrm{n} = -\frac{1}{2}:$$

$$= 2\sqrt{v}$$

Plug in solved integrals:

$$\frac{1}{\sin(a)} \int \frac{1}{\sqrt{v}}\, \mathrm{d}v$$

$$= \frac{2\sqrt{v}}{\sin(a)}$$

Undo substitution $v = \sin(a)u + \cos(a)$:

$$= \frac{2\sqrt{\sin(a)u + \cos(a)}}{\sin(a)}$$

Plug in solved integrals:

$$-\int \frac{1}{\sqrt{\sin(a)u+\cos(a)}}\,\mathrm{d}u$$

$$=-\frac{2\sqrt{\sin(a)u+\cos(a)}}{\sin(a)}$$

Undo substitution $u=\cot(x)$:

$$=-\frac{2\sqrt{\sin(a)\cot(x)+\cos(a)}}{\sin(a)}$$

The problem is solved:

$$\int \frac{1}{\sin^{\frac{3}{2}}(x)\sqrt{\sin(x+a)}}\,\mathrm{d}x$$

$$=-\frac{2\sqrt{\sin(a)\cot(x)+\cos(a)}}{\sin(a)}+C$$