$$\cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{8}\right)\cos\left(\frac{\pi}{16}\right)\cdots\cos\left(\frac{\pi}{2^{n-1}}\right)$$

now multiplying this with

$\frac{1}{2\sin\left(\frac{\pi}{2^{n-1}}\right)}$ then we get

$$\frac{1}{2\sin\left(\frac{\pi}{2^{n-1}}\right)}\left[\underbrace{2\cos\left(\frac{\pi}{2^{n-1}}\right)\sin\left(\frac{\pi}{2^{n-1}}\right)}\right]\cos\left(\frac{\pi}{2^{n-2}}\right)\cos\left(\frac{\pi}{2^{n-3}}\right)\cdots\cos\left(\frac{\pi}{8}\right)\cos\left(\frac{\pi}{4}\right)$$

$$\Rightarrow \frac{1}{2}\operatorname{cosec}\left(\frac{\pi}{2^{n-1}}\right)\left[\sin\left(\frac{2\pi}{2^{n-1}}\right)\right]\cos\left(\frac{\pi}{2^{n-2}}\right)\cdots\cos\left(\frac{\pi}{8}\right)\cos\left(\frac{\pi}{4}\right)$$

$$= \frac{1}{2}\operatorname{cosec}\left(\frac{\pi}{2^{n-1}}\right)\left[\sin\left(\frac{\pi}{2^{n-2}}\right)\right]\cos\left(\frac{\pi}{2^{n-2}}\right)\cdots\cos\left(\frac{\pi}{8}\right)\cos\left(\frac{\pi}{4}\right)$$

Again multipluplying and divide by 2

$$= \frac{1}{2^2}\operatorname{cosec}\left(\frac{\pi}{2^{n-1}}\right)\cdot\sin\left(\frac{\pi}{2^{n-3}}\right)\cos\left(\frac{\pi}{2^{n-3}}\right)\cdots\cos\left(\frac{\pi}{8}\right)\cos\left(\frac{\pi}{4}\right)$$

,
,
,
,

$$\frac{1}{2^{n-1}}\operatorname{cosec}\left(\frac{\pi}{2^n}\right)\sin\left(\frac{\pi}{2}\right)$$

$$= \frac{1}{2^{n-1}} \cosec\left(\frac{\pi}{2^n}\right)$$

now

$$\frac{\pi}{2} : \lim_{n\to\infty} \left\{ \cos\left(\frac{\pi}{4}\right) \cos\left(\frac{\pi}{8}\right) \cdots \cos\left(\frac{\pi}{2^{n-1}}\right) \right.$$

$$= \frac{\pi}{2} \cdot \lim_{n\to\infty} \frac{1}{2^{n-1}} \cosec\left(\frac{\pi}{2^n}\right)$$

$$= \pi \cdot \lim_{n\to\infty} \frac{1}{2^n} \left(\cosec\left(\pi/2^n\right)\right) =$$

Expansion of cosec x

$$\operatorname{cosec} x = \frac{1}{x} + \frac{x}{6} + \frac{7x^3}{360} + \frac{31x^5}{15120} + \cdots$$

$$\operatorname{cosec}\left(\frac{\pi}{2^n}\right) = \frac{2^n}{\pi} + \frac{\pi}{6\,2^n} + \frac{7\pi^3}{360 \cdot 2^{3n}} + \cdots$$

$$\frac{1}{2^n}\left(\operatorname{cosec}\left(\frac{\pi}{2^n}\right)\right) = \frac{1}{2^n}\left[\frac{2^n}{\pi} + \frac{\pi}{6\,2^n} + \frac{7\pi^3}{360 \cdot 2^{3n}} + \cdots\right]$$

$$= \frac{1}{\pi} + \underbrace{\frac{\pi}{6 \cdot 2^{2n}} + \frac{7\pi^3}{360\,2^{3n}} + \cdots}_{\downarrow\, 0}$$

$$\lim_{n \to \infty} \frac{1}{2^n}\left(\operatorname{cosec}\left(\frac{\pi}{2^n}\right)\right) = \frac{1}{\pi} + 0$$

$$= \frac{1}{\pi}$$

$$\therefore \lim_{n \to \infty}\left(\frac{1}{2^n} \operatorname{cosec}\left(\frac{\pi}{2^n}\right)\right) = \frac{1}{\pi}$$