$$\frac{d^2y}{dx^2} + \frac{2}{x}\frac{dy}{dx} + \frac{a^2}{x^4}y = 0 \quad -①$$

compairing equation ① with the standard equation

$$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$$

$$P = \frac{2}{x} \quad ; \quad Q = \frac{a^2}{x^4} \quad ; \quad R = 0$$

now by changing the independent variable x to z, the ~~eq~~ given equation becomes

$$\frac{d^2y}{dz^2} + P_1\frac{dy}{dz} + Q_1 y = R_1$$

now

$$P_1 = \frac{\frac{d^2z}{dx^2} + P\frac{dz}{dx}}{\left(\frac{dz}{dx}\right)^2} \quad ; \quad Q_1 = \frac{Q}{\left(\frac{dz}{dx}\right)^2} \quad ; \quad R_1 = \frac{R}{Q}$$

$$\therefore \quad Q_1 = \frac{Q}{\left(\frac{dz}{dx}\right)^2}$$

choose $Q_1 = a^2$

$$a^2 = \frac{a^2/x^4}{\left(\frac{dz}{dx}\right)^2}$$

$$\Rightarrow \left(\frac{dz}{dx}\right)^2 = \frac{1}{x^4} \quad \Rightarrow \quad \frac{dz}{dx} = \pm\frac{1}{x^2}$$

$$dz = \pm\frac{1}{x^2}\,dx \quad \Rightarrow \quad z = \mp\frac{1}{x}$$

$\therefore$ let $z = \frac{-1}{x}$

$$\frac{dz}{dx} = +\frac{1}{x^2}$$

$$\frac{d^2z}{dx^2} = \frac{-2}{x^3}$$

now $P_1 = \dfrac{-\frac{2}{x^3} + \left(\frac{2}{x}\right)\left(\frac{1}{x^2}\right)}{\left(\frac{dz}{dx}\right)^2} = \dfrac{0}{\left(\frac{dz}{dx}\right)} = 0$

$\Rightarrow P_1 = 0$

$Q_1 = a^2$

$R_1 = \dfrac{R}{Q} = \dfrac{0}{\left(a^2/x^4\right)} = 0$

hence $\dfrac{d^2y}{dz^2} + 0.\dfrac{dy}{dz} + a^2y = 0$

$\Rightarrow \dfrac{d^2y}{dz^2} + a^2y = 0$

$\Rightarrow (D^2 + a^2)y = 0$

Auxiliary equation $m^2 + a^2 = 0$

$\Rightarrow m = \pm ai$

C.F. $= C_1 \cos az + C_2 \sin az$

C.F. $= C_1 \cos\left(\frac{a}{x}\right) + C_2 \sin\left(\frac{a}{x}\right)$

P.I. $= 0$

~~$y(x) = c_1 \cos$~~

$$y(x) = c_1 \cos\left(\frac{a}{x}\right) + c_2 \sin\left(\frac{a}{x}\right)$$