$$f(x,y) = x^2 - xy - y + y^2$$

$$\nabla f = (2x-y)\hat{i} + (2y-1-x)\hat{j}$$

Direction derivative $= \left[(2x-y)\hat{i} + \hat{j}(2y-1-x)\right] \cdot \left[\frac{\hat{i}}{2} + \frac{\sqrt{3}\hat{j}}{2}\right]$

$$\Rightarrow \frac{2x-y}{2} + \frac{(2y-1-x)\sqrt{3}}{2} = 0$$

$$\Rightarrow 2x - y + 2\sqrt{3}y - \sqrt{3} - \sqrt{3}x = 0$$

put $x = 1$

$$2 - y + 2\sqrt{3}y - \sqrt{3} - \sqrt{3} = 0$$

$$y(2\sqrt{3} - 1) = 2\sqrt{3} - 2 \rightarrow y = \frac{2\sqrt{3} - 2}{2\sqrt{3} - 1}$$

option (c), (d) discarded

put $x = -1$

$$-2 - y + 2\sqrt{3}y - \sqrt{3} + \sqrt{3} = 0$$

$$y = \frac{2}{2\sqrt{3} - 1}$$

option (b) ✓