Problem:

$$\int \frac{1}{(x-2)^2\sqrt{x^2-4x+5}}\,\mathrm{d}x$$

Substitute $u = x - 2 \longrightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 1$ (steps) $\longrightarrow$

$\mathrm{d}x = \mathrm{d}u$, use:

$x^2 = (u+2)^2$

$$= \int \frac{1}{u^2\sqrt{u^2+1}}\,\mathrm{d}u$$

... or choose an alternative:

Don't substitute

Perform trigonometric substitution:

Substitute $u = \tan(v) \longrightarrow v = \arctan(u)$,

$\mathrm{d}u = \sec^2(v)\,\mathrm{d}v$ (steps):

$$= \int \frac{\sec^2(v)}{\tan^2(v)\sqrt{\tan^2(v)+1}}\,\mathrm{d}v$$

Simplify using $\tan^2(v) + 1 = \sec^2(v)$:

$$= \int \frac{\sec(v)}{\tan^2(v)}\,\mathrm{d}v$$

... or choose an alternative:

Perform hyperbolic substitution

... or choose an alternative:

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Rewrite/simplify using trigonometric/hyperbolic identities:

$$= \int \frac{\cos(v)}{\sin^2(v)}\,\mathrm{d}v$$

Substitute $w = \sin(v) \longrightarrow \frac{\mathrm{d}w}{\mathrm{d}v} = \cos(v)$ (steps) $\longrightarrow$ $\mathrm{d}v = \frac{1}{\cos(v)}\,\mathrm{d}w$:

$$= \int \frac{1}{w^2}\,\mathrm{d}w$$

Apply power rule:

$\int w^{\mathrm{n}}\,\mathrm{d}w = \frac{w^{\mathrm{n}+1}}{\mathrm{n}+1}$ with $\mathrm{n} = -2$:

$$= -\frac{1}{w}$$

Undo substitution $w = \sin(v)$:

$$= -\frac{1}{\sin(v)}$$

Undo substitution $v = \arctan(u)$, use:

$\sin(\arctan(u)) = \frac{u}{\sqrt{u^2+1}}$

$$= -\frac{\sqrt{u^2+1}}{u}$$

Undo substitution $u = x - 2$:

$$= -\frac{\sqrt{(x-2)^2+1}}{x-2}$$

The problem is solved:

$$\int \frac{1}{(x-2)^2\sqrt{x^2-4x+5}}\,\mathrm{d}x$$

$$= -\frac{\sqrt{(x-2)^2+1}}{x-2} + C$$

ANTIDERIVATIVE COMPUTED BY MAXIMA:

$$\int \boldsymbol{f(x)}\,\mathbf{d}\boldsymbol{x} = \boldsymbol{F(x)} =$$

$$-\frac{\sqrt{x^2-4x+5}}{x-2} + C$$