$$x \frac{d}{dx}\left(x\frac{dy}{dx} - y\right) - 2x\frac{dy}{dx} + 2y + x^2 y = 0$$

$$\Rightarrow x\frac{d}{dx}\left(x\frac{dy}{dx}\right) - x\frac{dy}{dx} - 2x\frac{dy}{dx} + 2y + x^2y = 0$$

$$\Rightarrow x\left[\frac{d(x)}{dx}\cdot\frac{dy}{dx} + x\frac{d}{dx}\left(\frac{dy}{dx}\right)\right] - 3x\frac{dy}{dx} + 2y + x^2y = 0$$

$$\Rightarrow x\left[\cdot\frac{dy}{dx} + x\frac{d^2y}{dx^2}\right] - 3x\frac{dy}{dx} + 2y + x^2y = 0$$

$$\Rightarrow x\frac{dy}{dx} + x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 2y + x^2y = 0$$

$$\Rightarrow x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + (2 + x^2)y = 0$$

$$\Rightarrow \frac{d^2y}{dx^2} - \frac{2}{x}\frac{dy}{dx} + \left(1 + \frac{2}{x^2}\right)y = 0 \quad (1)$$

now compairing eqn (1) with

$$\frac{d^2y}{dx^2} + p\frac{dy}{dx} + Qy = R$$

we get $P = -\frac{2}{x}$; $Q = 1 + \frac{2}{x^2}$; $R = 0$

To remove the first derivation

$$u = e^{-\frac{1}{2}\int P\,dx} = e^{-\frac{1}{2}\int\left(\frac{-2}{x}\right)dx}$$

$u = e^{\int \frac{dx}{x}} = e^{\log x} = x$

$u = x$

and $I = Q - \frac{1}{4}P^2 - \frac{1}{2}\frac{dP}{dx}$

$I = \left(1 + \frac{2}{x^2}\right) - \frac{1}{4}\left(\frac{4}{x^2}\right) - \frac{1}{2}\left(\frac{2}{x^2}\right)$

$= 1 + \frac{2}{x^2} - \frac{1}{x^2} - \frac{1}{x^2}$

$I = 1$

$S = R e^{\frac{1}{2}\int P dx} = 0 \quad (\because R = 0$

$\therefore$ The normal form

$\frac{d^2v}{dx^2} + Iv = S$ of the given eqn

$\frac{d^2v}{dx^2} + 1 \cdot v = 0$

$\Rightarrow (D^2+1)v = 0$

A.E (Auxiliary equation)

$m^2 + 1 = 0$

$\Rightarrow m = \pm i$ that are complex

C.F. $= c_1 \cos x + c_2 \sin x$

and P.I. $= 0$

hence $v = C.F. + P.I$

$= c_1 \cos x + c_2 \sin x$

so complete solution is

$y = vu$

$y = x(c_1 \cos x + c_2 \sin x)$