

$$y = v e^{a \sin^{-1} x}$$

where v is function of x

$$y = v e^{a \sin^{-1} x}$$

$$\frac{dy}{dx} = \frac{d}{dx} \left(v e^{a \sin^{-1} x} \right)$$

$$= \frac{dv}{dx}$$

using chain rule $d(u \cdot v) = u \cdot dv + v \cdot du$

$$= v \cdot \frac{d}{dx} (e^{a \sin^{-1} x}) + e^{a \sin^{-1} x} \cdot \frac{dv}{dx}$$

$$\frac{dy}{dx} = v \cdot a e^{a \sin^{-1} x} \cdot \frac{d(\sin^{-1} x)}{dx} + e^{a \sin^{-1} x} \cdot \frac{dv}{dx}$$

$$\therefore \frac{d(e^{ax})}{dx} = e^{ax} \cdot \frac{d(ax)}{dx} = a e^{ax}$$

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$$\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{dy}{dx} = a e^{a \sin^{-1}x} \cdot \frac{1}{\sqrt{1-x^2}} + e^{a \sin^{-1}x} \frac{dv}{dx}$$

$$\frac{dy}{dx} = \frac{a}{\sqrt{1-x^2}} e^{a \sin^{-1}x} + e^{a \sin^{-1}x} \frac{dv}{dx} \quad \text{--- (1)}$$

Now again differentiating eqn (1) with respect to x we get

$$\frac{d^2y}{dx^2} = \frac{d}{dx} \left(\frac{dy}{dx} \right) = \frac{d}{dx} \left[\frac{a}{\sqrt{1-x^2}} e^{a \sin^{-1}x} + e^{a \sin^{-1}x} \frac{dv}{dx} \right]$$

$$\therefore d(u+v) = du + dv$$

$$= \underbrace{\frac{d}{dx} \left(\frac{a}{\sqrt{1-x^2}} \right)}_I + \underbrace{\frac{d}{dx} \left(e^{a \sin^{-1}x} \frac{dv}{dx} \right)}_{II}$$

Again using chain rule for differentiating

$$= \frac{a}{\sqrt{1-x^2}} \cdot \frac{d(e^{a \sin^{-1}x})}{dx} + e^{a \sin^{-1}x} \cdot \frac{d}{dx} \left(\frac{a}{\sqrt{1-x^2}} \right)$$

$$+ e^{a \sin^{-1}x} \cdot \frac{d}{dx} \left(\frac{dv}{dx} \right) + \frac{dv}{dx} \cdot \frac{d}{dx} (e^{a \sin^{-1}x})$$

$$= \frac{a}{\sqrt{1-x^2}} e^{a \sin^{-1} x} \cdot \frac{d(a \sin^{-1} x)}{dx} + e^{a \sin^{-1} x} \cdot a \frac{d}{dx} \left(\frac{1}{\sqrt{1-x^2}} \right) \\ + e^{a \sin^{-1} x} \cdot \frac{d^2 v}{dx^2} + \frac{dv}{dx} \cdot e^{a \sin^{-1} x} \cdot \frac{d(a \sin^{-1} x)}{dx}$$

Now

$$\frac{d(a \sin^{-1} x)}{dx} = a \frac{d(\sin^{-1} x)}{dx} = a \cdot \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} \left(\frac{1}{\sqrt{1-x^2}} \right) = \frac{d}{dx} (1-x^2)^{-\frac{1}{2}} = -\frac{1}{2} (1-x^2)^{-\frac{1}{2}-1} \cdot \frac{d(-x^2)}{dx}$$

$$= -\frac{1}{2} \cdot \frac{1}{(1-x^2)^{3/2}} \cdot (-2x) = \frac{x}{(1-x^2)^{3/2}}$$

$$\frac{d}{dx} \left(\frac{1}{\sqrt{1-x^2}} \right) = \frac{x}{(1-x^2)^{3/2}}$$

So

$$= \frac{a e^{a \sin^{-1} x}}{\sqrt{1-x^2}} \cdot \frac{a}{\sqrt{1-x^2}} + e^{a \sin^{-1} x} \cdot \frac{a x}{(1-x^2)^{3/2}} + e^{a \sin^{-1} x} \cdot \frac{d^2 v}{dx^2} \\ + \frac{dv}{dx} e^{a \sin^{-1} x} \cdot \frac{a}{\sqrt{1-x^2}}$$

$$\frac{d^2 y}{dx^2} = \frac{a^2 e^{a \sin^{-1} x}}{(1-x^2)} + \frac{a x}{(1-x^2)^{3/2}} e^{a \sin^{-1} x} + e^{a \sin^{-1} x} \frac{d^2 v}{dx^2} \\ + \frac{a e^{a \sin^{-1} x}}{\sqrt{1-x^2}} \frac{dv}{dx}$$