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$$\frac{d^2x}{dt^2} + \frac{dx}{dt} - \beta x = 0$$

A.E $\quad m^2 + m - \beta = 0$

$$m = \frac{-1 \pm \sqrt{1+4\beta}}{2}$$

$$m_1 = \frac{-1+\sqrt{1+4\beta}}{2}, \quad m_2 = \frac{-1-\sqrt{1+4\beta}}{2}$$

~~Hence and~~

So C.F. $= c_1 e^{m_1 t} + c_2 e^{m_2 t}$

$$= c_1 e^{\left(\frac{-1+\sqrt{1+4\beta}}{2}\right)t} + c_2 e^{\left(\frac{-1-\sqrt{1+4\beta}}{2}\right)t}$$

and P.I $= 0$

$x(t) =$ C.F + P.I

$$x(t) = c_1 e^{\left(\frac{-1+\sqrt{4\beta+1}}{2}\right)t} + c_2 e^{\left(\frac{-1-\sqrt{1+4\beta}}{2}\right)t}$$

$$x(t) = c_1 e^{-\left(\frac{1-\sqrt{4\beta+1}}{2}\right)t} + c_2 e^{-\left(\frac{1+\sqrt{1+4\beta}}{2}\right)t}$$

$\because$ as $t \to \infty \quad x(t) \to 0$

$\because \lim_{t\to\infty} e^{-t} = 0$

So

$$x(t) = c_1 e^{-\frac{(1-\sqrt{1+4\beta})}{2}t} + c_2 e^{-\frac{(1+\sqrt{1+4\beta})}{2}t}$$

this will be zero if

this is always will be zero as $t \to \infty$

$$\frac{1-\sqrt{1+4\beta}}{2} > 0$$

$$\Rightarrow 1 - \sqrt{1+4\beta} > 0$$

$$\Rightarrow \sqrt{1+4\beta} < 1$$

$$1+4\beta < 0$$

$$\beta < 0 \quad \Rightarrow \quad \beta > -\frac{\pi}{2}$$