Problem:

$$\int \left((\sin(2x) + 1)^{\frac{3}{2}} - 1 \right) \mathrm{d}x$$

Apply linearity:

$$= \int (\sin(2x) + 1)^{\frac{3}{2}} \,\mathrm{d}x - \int 1 \,\mathrm{d}x$$

Now solving:

$$\int (\sin(2x) + 1)^{\frac{3}{2}} \,\mathrm{d}x$$

Substitute $u = 2x \longrightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 2$ (steps) $\longrightarrow$ $\mathrm{d}x = \frac{1}{2}\,\mathrm{d}u$:

$$= \frac{1}{2} \int (\sin(u) + 1)^{\frac{3}{2}} \,\mathrm{d}u$$

Now solving:

$$\int (\sin(u) + 1)^{\frac{3}{2}} \,\mathrm{d}u$$

Rewrite/simplify using trigonometric/hyperbolic identities:

$$= \int 2^{\frac{3}{2}} \cos^3\left(\frac{2u - \pi}{4}\right) \mathrm{d}u$$

Apply linearity:

$$= 2^{\frac{3}{2}} \int \cos^3\left(\frac{2u - \pi}{4}\right) \mathrm{d}u$$

Now solving:

$$\int \cos^3\left(\frac{2u-\pi}{4}\right) \mathrm{d}u$$

Substitute $v = \frac{2u-\pi}{4} \longrightarrow \frac{\mathrm{d}v}{\mathrm{d}u} = \frac{1}{2}$ (steps) $\longrightarrow$ $\mathrm{d}u = 2\,\mathrm{d}v$:

$$= 2\int \cos^3(v)\,\mathrm{d}v$$

Now solving:

$$\int \cos^3(v)\,\mathrm{d}v$$

Prepare for substitution:

$$= \int \cos(v)\left(1-\sin^2(v)\right)\,\mathrm{d}v$$

Substitute $w = \sin(v) \longrightarrow \frac{\mathrm{d}w}{\mathrm{d}v} = \cos(v)$ (steps) $\longrightarrow$ $\mathrm{d}v = \frac{1}{\cos(v)}\,\mathrm{d}w$:

$$= \int \left(1-w^2\right)\mathrm{d}w$$

Apply linearity:

$$= \int 1\,\mathrm{d}w - \int w^2\,\mathrm{d}w$$

Now solving:

$$\int 1\,\mathrm{d}w$$

Apply constant rule:

$$= w$$

Now solving:

$$\int 1\,\mathrm{d}w$$

Apply constant rule:

$$= w$$

Now solving:

$$\int w^2\,\mathrm{d}w$$

Apply power rule:

$$\int w^{\mathrm{n}}\,\mathrm{d}w = \frac{w^{\mathrm{n}+1}}{\mathrm{n}+1} \text{ with } \mathrm{n} = 2:$$

$$= \frac{w^3}{3}$$

Plug in solved integrals:

$$\int 1\,\mathrm{d}w - \int w^2\,\mathrm{d}w$$

$$= w - \frac{w^3}{3}$$

Undo substitution $w = \sin(v)$:

$$= \sin(v) - \frac{\sin^3(v)}{3}$$

Plug in solved integrals:

$$2\int \cos^3(v)\,\mathrm{d}v$$

$$= 2\sin(v) - \frac{2\sin^3(v)}{3}$$

Undo substitution $v = \frac{2u - \pi}{4}$:

$$= 2\sin(v) - \frac{2\sin^3(v)}{3}$$

Undo substitution $v = \frac{2u-\pi}{4}$:

$$= 2\sin\left(\frac{2u-\pi}{4}\right) - \frac{2\sin^3\left(\frac{2u-\pi}{4}\right)}{3}$$

Plug in solved integrals:

$$2^{\frac{3}{2}} \int \cos^3\left(\frac{2u-\pi}{4}\right) du$$

$$= 2^{\frac{5}{2}} \sin\left(\frac{2u-\pi}{4}\right) - \frac{2^{\frac{5}{2}} \sin^3\left(\frac{2u-\pi}{4}\right)}{3}$$

Plug in solved integrals:

$$\frac{1}{2} \int (\sin(u) + 1)^{\frac{3}{2}} du$$

$$= 2^{\frac{3}{2}} \sin\left(\frac{2u-\pi}{4}\right) - \frac{2^{\frac{3}{2}} \sin^3\left(\frac{2u-\pi}{4}\right)}{3}$$

Undo substitution $u = 2x$:

$$= 2^{\frac{3}{2}} \sin\left(\frac{4x-\pi}{4}\right) - \frac{2^{\frac{3}{2}} \sin^3\left(\frac{4x-\pi}{4}\right)}{3}$$

Now solving:

$$\int 1 \, dx$$

Use previous result:

$$= x$$

$$\int (\sin(2x)+1)^{\frac{3}{2}}\,\mathrm{d}x - \int 1\,\mathrm{d}x$$

$$= -\frac{2^{\frac{3}{2}}\sin^3\left(\frac{4x-\pi}{4}\right)}{3} + 2^{\frac{3}{2}}\sin\left(\frac{4x-\pi}{4}\right) - x$$

The problem is solved:

$$\int \left((\sin(2x)+1)^{\frac{3}{2}} - 1\right)\mathrm{d}x$$

$$= -\frac{2^{\frac{3}{2}}\sin^3\left(\frac{4x-\pi}{4}\right)}{3} + 2^{\frac{3}{2}}\sin\left(\frac{4x-\pi}{4}\right) - x + C$$

Rewrite/simplify:

$$= -\frac{\sin(3x)+\cos(3x)-9\sin(x)+9\cos(x)+6x}{6} + C$$