The given line is $\frac{x-13}{5} = \frac{y+8}{-8} = \frac{z-31}{1} = \lambda$ (say) — (1)

$\Rightarrow \frac{x-13}{5} = \lambda \Rightarrow x-13 = 5\lambda \Rightarrow x = 13+5\lambda$

$\Rightarrow \frac{y+8}{-8} = \lambda \Rightarrow y+8 = -8\lambda \Rightarrow y = -8\lambda - 8$

$\Rightarrow \frac{z-31}{1} = \lambda \Rightarrow z-31 = \lambda \Rightarrow z = \lambda + 31$

Let N be the foot of the perpendicular from point $P(1,3,-1)$ to the given line

The the coordinate of $N = (5\lambda+13, -8\lambda-8, \lambda+31)$ for some values of λ

direction ratios of PN are $(5\lambda+13-1), (-8\lambda-8-3), (\lambda+31-(-1))$

$= (5\lambda+12, -8\lambda-11, \lambda+32)$

direction ratios of given line are $5, -8, 1$

$\because$ $PN \perp$ to the given line (1), we have

$5(5\lambda+12) + (-8)(-8\lambda-11) + 1(\lambda+32) = 0$

$\Rightarrow 25\lambda + 60 + 64\lambda + 88 + \lambda + 32 = 0$

$\Rightarrow 90\lambda + 180 = 0$

$\lambda = -2$

Hence the ~~direction ra~~ coordinate of N are

$(5(-2)+13, -8(-2)+8, -2+31)$

$= (3, 8, 29)$

$$\therefore PN = \sqrt{(3-1)^2 + (8-3)^2 + (29+1)^2}$$

$$= \sqrt{4 + 25 + 900}$$

$$= \sqrt{929} \text{ unit}$$