

$$0 < a_1 < b_1$$

We know

$$A.M \geq G.M$$

$$\Rightarrow \frac{a_n + b_n}{2} \geq \sqrt{a_n \cdot b_n}$$

$$\Rightarrow b_{n+1} > a_{n+1} \quad \forall n \geq 1$$

Now

$$\Rightarrow a_{n+1} = \sqrt{a_n \cdot b_n} > \sqrt{a_n^2} = a_n$$

$$\Rightarrow a_{n+1} > a_n \quad \forall n \geq 1$$

$\Rightarrow \{a_n\}$ is monotonically increasing and bounded above by b_2

$$\text{and } b_{n+1} = \frac{a_n + b_n}{2} < \frac{b_n + b_n}{2} = b_n$$

$$\Rightarrow b_{n+1} < b_n$$

$\Rightarrow \{b_n\}$ is monotonically decreasing and bounded below by a_1 .

So, $\{a_n\}, \{b_n\}$ is convergent.

Now let $\lim a_n = l_1$ & $\lim b_n = l_2$

$$\text{So } \lim a_{n+1} = \lim \sqrt{a_n \cdot b_n}$$

$$l_1 = \sqrt{l_1 \cdot l_2} \Rightarrow l_1 = l_2$$

So, option (b) is correct