: The series is $\sum_{n=3}^{\infty} \frac{a^n}{n^b (\log_e n)^c}$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{a^{n+1}}{(n+1)^b (\log_e (n+1))^c} \cdot \frac{n^b (\log_e n)^c}{a^n}\right|$$

$$= |a|$$

Series is convergent if $|a| < 1, b \in \mathbb{R}, c \in \mathbb{R}$

Option (b):

$$\sum_{n=3}^{\infty} \frac{1}{n^b (\log_e n)^c}$$

By Cauchy condensation test

$$\sum_{n=3}^{\infty} \frac{2^n}{2^{nb} (n \log_e 2)^c}$$

$$= \sum_{n=3}^{\infty} \frac{1}{2^{n(b-1)} n^c \cdot (\log_e 2)^c}$$

$$= \sum b_n \quad \text{(let)}$$

Now, $\lim_{n\to\infty} \left| \frac{b_{n+1}}{b_n} \right| = \lim_{n\to\infty} \frac{2^{n(b-1)} n^c (\log_e 2)^c}{2^{(n+1)(b-1)} (n+1)^c (\log_e 2)^c}$

$$= \frac{1}{2^{b-1}} < 1$$

$\Rightarrow$ $\sum b_n$ is convergent

$\Rightarrow$ Given series is convergent

Option (c):

By Cauchy condensation test,

Option (c) does not result in convergence because case when $b < 1$, series becomes divergent.

Option (d):

By Dirichlet test,

Option (d) is true.