$$f(x,y) = \begin{cases} \dfrac{x^2 y}{x^4 + y^2} \\ 0 \end{cases}$$

Directional derivative $= \lim_{h \to 0} \dfrac{f(a+hu) - f(a)}{h}$

$$\Rightarrow \lim_{h \to 0} \frac{f\left((0,0) + h\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)\right) - f(0,0)}{h}$$

$$\Rightarrow \lim_{h \to 0} \frac{f\left(\frac{h}{\sqrt{2}}, \frac{h}{\sqrt{2}}\right) - f(0,0)}{h}$$

$$\Rightarrow \lim_{h \to 0} \frac{\frac{h^3}{2\sqrt{2}}}{h\left(\frac{h^4}{4} + \frac{h^2}{2}\right)} \Rightarrow \frac{\frac{h^3}{2\sqrt{2}}}{h\left(\frac{h^4 + 2h^2}{4}\right)}$$

$$\Rightarrow \lim_{h \to 0} \frac{h^{\cancel{3}\,2}}{\cancel{4}\sqrt{2}} \times \frac{\cancel{4}\,2}{\cancel{h}(h^4 + 2h^2)}$$

$$\Rightarrow \lim_{h \to 0} \frac{2h^2}{\sqrt{2}(h^4 + 2h^2)} \Rightarrow \lim_{h \to 0} \frac{2}{\sqrt{2}(h^2 + 2)}$$

$$\frac{2}{\sqrt{2} \times 2} = \boxed{\frac{1}{\sqrt{2}}}$$