

$$f(x,y) = \begin{cases} \frac{x^2 y}{x^4 + y^2} \\ 0 \end{cases}$$

Directional derivative = $\lim_{h \rightarrow 0} \frac{f(a+hw) - f(a)}{h}$

$$\Rightarrow \lim_{h \rightarrow 0} \frac{f\left((0,0) + h\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)\right) - f(0,0)}{h}$$

$$\Rightarrow \lim_{h \rightarrow 0} \frac{f\left(\frac{h}{\sqrt{2}}, \frac{h}{\sqrt{2}}\right) - f(0,0)}{h}$$

$$\Rightarrow \lim_{h \rightarrow 0} \frac{\cancel{h^3} \frac{h^3}{2\sqrt{2}}}{h \left(\frac{h^4}{4} + \frac{h^2}{2} \right)} \quad \text{or} \quad \frac{h^3}{2\sqrt{2}}$$

$$\Rightarrow \lim_{h \rightarrow 0} \frac{h^{\frac{3}{2}}}{2\sqrt{2}} \times \frac{4^{\frac{1}{2}}}{h(h^4 + 2h^2)}$$

$$\Rightarrow \lim_{h \rightarrow 0} \frac{2h^2}{\sqrt{2}(h^4 + 2h^2)} \Rightarrow \lim_{h \rightarrow 0} \frac{2}{\sqrt{2}(h^2 + 2)}$$

$$= \boxed{\infty}$$

not exist