$$f(x+y) = 2f(x) + xf(x) + y\sqrt{f(x)} \quad \text{— (i)}$$

We know $\quad f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$$\Rightarrow \cancel{2f(x)+} \; f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x+0)}{h}$$

$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\cancel{2f(x)} + \cancel{xf(x)} + h\sqrt{f(x)} - \cancel{2f(x)} - \cancel{xf(x)} - 0}{h}$$

$$= \lim_{h \to 0} \sqrt{f(x)} = \sqrt{f(x)}$$

$$\boxed{f'(x) = \sqrt{f(x)}} \Rightarrow$$

$f(x)''$

$$\int \frac{f'(x)}{\sqrt{f(x)}} = \int 1$$

$$\text{let } f(x) = t \Rightarrow \frac{dt}{dx} = f'(x)$$

$$\Rightarrow \int \frac{\cancel{f'(x)}}{\sqrt{t}} \times \frac{dt}{\cancel{f'(x)}} = \int 1$$

$$\Rightarrow \quad 2\sqrt{t} = x + c$$

$$2\sqrt{f(x)} = x + c$$

$$\boxed{f(x) = \left(\frac{x}{2} + c\right)^2}$$

$$f'(0) = 0 \Rightarrow \quad f'(x) = 2\left(\frac{x}{2} + c\right) \times \frac{1}{2}$$

$$f'(0) = 0 = \frac{0}{2} + c \Rightarrow \boxed{c = 0}$$

$$f(x) = \left(\frac{x}{2}\right)^2 \Rightarrow f(6) = \left(\frac{6}{2}\right)^2 = \boxed{9} \text{ Ans}$$