

Problem:

$$\int \frac{x^2}{(x \sin(x) + \cos(x))^2} dx$$

Rewrite:

$$= \int \left(\frac{x \sin(x)}{x \sin(x) + \cos(x)} - \frac{x \cos(x) (\sin(x) - x \cos(x))}{(x \sin(x) + \cos(x))^2} \right) dx$$

Apply linearity:

$$= \int \frac{x \sin(x)}{x \sin(x) + \cos(x)} dx - \int \frac{x \cos(x) (\sin(x) - x \cos(x))}{(x \sin(x) + \cos(x))^2} dx$$

Within the sum, apply integration by parts for $\int \frac{x \cos(x) (\sin(x) - x \cos(x))}{(x \sin(x) + \cos(x))^2} dx$: $\int \mathbf{f} \mathbf{g}' = \mathbf{f} \mathbf{g} - \int \mathbf{f}' \mathbf{g}$,

$$\mathbf{f} = \sin(x) - x \cos(x), \mathbf{g}' = \frac{x \cos(x)}{(x \sin(x) + \cos(x))^2}$$

$\downarrow$ steps

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$$\mathbf{f}' = x \sin(x), \quad \mathbf{g} = -\frac{1}{x \sin(x) + \cos(x)};$$

$$= \frac{\sin(x) - x \cos(x)}{x \sin(x) + \cos(x)} + \int -\frac{x \sin(x)}{x \sin(x) + \cos(x)} dx + \int \frac{x \sin(x)}{x \sin(x) + \cos(x)} dx$$

The integral $\int \frac{x \sin(x)}{x \sin(x) + \cos(x)} dx$ cancels, leaving only:

$$= \frac{\sin(x) - x \cos(x)}{x \sin(x) + \cos(x)}$$

ANTIDERIVATIVE COMPUTED BY MAXIMA:

$$\int f(x) \, dx = F(x) =$$

$$\frac{(2x^2 - 2) \sin(2x) + 4x \cos(2x)}{(x^2 + 1) \sin^2(2x) + 4x \sin(2x) + (x^2 + 1) \cos^2(2x) + (2 - 2x^2) \cos(2x) + x^2 + 1} + C$$

Simplify