$$f(x) = (1+b^2)x^2 + 2bx + 1$$

$$f'(x) = 2(1+b^2)x + 2b = 0$$

$$x = \frac{-b}{(1+b^2)}$$

$$f''(x) = 2(1+b^2) > 0$$

$\Rightarrow$ $x = \frac{-b}{1+b^2}$ is minimin (local)

$$f\left(\frac{-b}{1+b^2}\right) = m(b) = (1+b^2)\left(\frac{b^2}{(1+b^2)^2}\right) + 2b\left(\frac{-b}{1+b^2}\right) + 1$$

$\Rightarrow$ $\frac{1}{1+b^2}$

As b varies

$m(b)$ lies in $(0,1]$

Option (4)