

Date

since region is closed, use divergence theorem.

$$\iiint F \cdot n \, dS = \iiint \nabla F \, dV$$

$$= \int_{x=-1}^1 \int_{y=0}^{1+x^2} \int_{z=0}^{1-x^2} (y + 2y) \, dz \, dy \, dx$$

$$\Rightarrow 3 \int_{x=-1}^1 \int_{y=0}^{1+x^2} y(1-x^2) \, dy \, dx \quad \begin{cases} 2-y=1-x^2 \\ y=1+x^2 \end{cases}$$

$$\Rightarrow \frac{3}{2} \int_{x=-1}^1 (1-x^2) [y^2]_0^{1+x^2} \, dx$$

$$\Rightarrow \frac{3}{2} \int_{-1}^1 (1-x^2)(1+x^2)^2 \, dx$$

$$\Rightarrow \frac{3}{2} \int_{x=-1}^1 (1-x^2)(1+x^4+2x^2) \, dx$$

$$\Rightarrow \frac{3}{2} \int_{x=-1}^1 (1+x^4+2x^2-x^2-x^6-2x^4) \, dx$$

$$\Rightarrow \frac{3}{2} \int_{x=-1}^1 (1-x^4+x^2-x^6) \, dx$$

$$\Rightarrow \frac{3}{2} \left[x - \frac{x^5}{5} + \frac{x^3}{3} - \frac{x^7}{7} \right]_{-1}^1 \Rightarrow \frac{3}{2} \left[1 - \frac{1}{5} + \frac{1}{3} - \frac{1}{7} \right]$$