$$f(x,y) = |y-2|\sqrt{|x-1|}$$

option (a) at $x=1$ $\sqrt{|x-1|}$ is not differentiable

$\Rightarrow$ $f(x,y)$ is not differentiable at $(1,2)$

✗

option (b) Yes, $f(x,y)$ is continuous as modulus function always continuous

option (c)

$$f_x(1,2) = \lim_{h\to 0} \frac{f(1+h,2) - f(1,2)}{h}$$

$$= \frac{0-0}{h} = 0 \quad \text{exist}$$

option (d)

Directional derivative $= \lim_{h\to 0} \frac{f((1,2)+h\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)) - f(1,2)}{h}$

$$\lim_{h\to 0} \frac{f\left(1+\frac{h}{\sqrt{2}}, 2+\frac{h}{\sqrt{2}}\right) - f(1,2)}{h}$$

$$\lim_{h\to 0} \frac{\left|\frac{h}{\sqrt{2}}\right|\sqrt{\left|\frac{h}{\sqrt{2}}\right|} - 0}{h}$$

→ L.H.L $\lim \frac{h^{3/2}}{h(2)^{3/4}} = 0$ $\neq 1$

→ 0

✗