

$$\nabla \times F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ z^2 & y^2 & x \end{vmatrix} = \hat{i}(0-0) - \hat{j}(1-2z) + \hat{k}(0) = (2z-1)\hat{j}$$

Use Stoke's theorem

$$\int F \cdot d\mathbf{r} = \iint (\nabla \times F) \cdot \hat{n} \, dS$$

$$= \iint_A \frac{2z-1}{\sqrt{3}} \times \frac{1}{\sqrt{3}} \, dS$$

$$\Rightarrow \iint 2(1-x-y)-1 \, dx \, dy$$

$$\Rightarrow \iint 1-2(x+y)$$

$$\Rightarrow \iint 1 - 2 \iint x+y \Rightarrow \frac{1}{2} \times 1 \times 1 - 2 \int_{x=0}^1 \int_{y=0}^{1-x} x+y \, dy \, dx$$

$$\Rightarrow \frac{1}{2} - 2 \int_{x=0}^1 \left(xy + \frac{y^2}{2} \right) \Big|_0^{1-x} dx \Rightarrow \frac{1}{2} - 2 \int_{x=0}^1 x(1-x) + \frac{(1-x)^2}{2} \, dx$$

$$\Rightarrow \frac{1}{2} - 2 \int_{x=0}^1 \frac{2x-2x^2+1+x^2-2x}{2} \, dx \Rightarrow \frac{1}{2} - \int_{x=0}^1 1-x^2 \, dx$$

$$\Rightarrow \frac{1}{2} - \left[x - \frac{x^3}{3} \right]_0^1 \Rightarrow \frac{1}{2} - \left[1 - \frac{1}{3} \right]$$

$$\Rightarrow \frac{1}{2} - \left[\frac{2}{3} \right] = \frac{3-4}{6} \Rightarrow \left[-\frac{1}{6} \right] \text{ Ans}$$

