Problem:

$$\int \frac{1}{(\sin(x)-1)(\sin(x)+4)}\,\mathrm{d}x$$

Prepare for tangent half-angle substitution (Weierstrass substitution):

$$= \int \frac{1}{\left(\frac{2\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}-1\right)\left(\frac{2\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}+4\right)}\,\mathrm{d}x$$

Substitute $u = \tan\left(\frac{x}{2}\right) \longrightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{\sec^2\left(\frac{x}{2}\right)}{2}$ (steps)

$\longrightarrow \mathrm{d}x = \frac{2}{\sec^2\left(\frac{x}{2}\right)}\,\mathrm{d}u = \frac{2}{u^2+1}\,\mathrm{d}u$:

$$= -\int \frac{u^2+1}{(u^2-2u+1)(2u^2+u+2)}\,\mathrm{d}u$$

Now solving:

$$\int \frac{u^2+1}{(u^2-2u+1)(2u^2+u+2)}\,\mathrm{d}u$$

Factor the denominator:

$$= \int \frac{u^2+1}{(u-1)^2(2u^2+u+2)}\,\mathrm{d}u$$

Perform partial fraction decomposition:

$$= \int \left(\frac{1}{5(2u^2+u+2)} + \frac{2}{5(u-1)^2}\right)\mathrm{d}u$$

Apply linearity:

$$= \frac{1}{5}\int \frac{1}{2u^2+u+2}\,\mathrm{d}u + \frac{2}{5}\int \frac{1}{(u-1)^2}\,\mathrm{d}u$$

Now solving:

$$\int \frac{1}{2u^2+u+2}\,\mathrm{d}u$$

Complete the square:

$$= \int \frac{1}{\left(\sqrt{2}u + \frac{1}{2^{\frac{3}{2}}}\right)^2 + \frac{15}{8}}\,\mathrm{d}u$$

Substitute $v = \dfrac{4u+1}{\sqrt{15}} \longrightarrow \dfrac{\mathrm{d}v}{\mathrm{d}u} = \dfrac{4}{\sqrt{15}}$ (steps) $\longrightarrow$ $\mathrm{d}u = \dfrac{\sqrt{15}}{4}\,\mathrm{d}v$:

$$= \int \frac{2\sqrt{15}}{15v^2+15}\,\mathrm{d}v$$

Simplify:

$$= \frac{2}{\sqrt{15}} \int \frac{1}{v^2+1}\,\mathrm{d}v$$

Now solving:

$$\int \frac{1}{v^2+1}\,\mathrm{d}v$$

This is a standard integral:

$$= \arctan(v)$$

Plug in solved integrals:

$$\frac{2}{\sqrt{15}}\int\frac{1}{v^2+1}\,\mathrm{d}v$$

$$=\frac{2\arctan(v)}{\sqrt{15}}$$

Undo substitution $v=\dfrac{4u+1}{\sqrt{15}}$:

$$=\frac{2\arctan\left(\frac{4u+1}{\sqrt{15}}\right)}{\sqrt{15}}$$

Now solving:

$$\int\frac{1}{(u-1)^2}\,\mathrm{d}u$$

Substitute $v=u-1\longrightarrow\dfrac{\mathrm{d}v}{\mathrm{d}u}=1$ (steps) $\longrightarrow$ $\mathrm{d}u=\mathrm{d}v$:

$$=\int\frac{1}{v^2}\,\mathrm{d}v$$

Apply power rule:

$\displaystyle\int v^{\mathrm{n}}\,\mathrm{d}v=\frac{v^{\mathrm{n}+1}}{\mathrm{n}+1}$ with $\mathrm{n}=-2$:

$$=-\frac{1}{v}$$

Undo substitution $v=u-1$:

$$=-\frac{1}{u-1}$$

Plug in solved integrals:

$$\frac{1}{5}\int \frac{1}{2u^2+u+2}\,du + \frac{2}{5}\int \frac{1}{(u-1)^2}\,du$$

$$= \frac{2\arctan\left(\frac{4u+1}{\sqrt{15}}\right)}{5\sqrt{15}} - \frac{2}{5(u-1)}$$

Plug in solved integrals:

$$-\int \frac{u^2+1}{(u^2-2u+1)(2u^2+u+2)}\,du$$

$$= \frac{2}{5(u-1)} - \frac{2\arctan\left(\frac{4u+1}{\sqrt{15}}\right)}{5\sqrt{15}}$$

Undo substitution $u = \tan\left(\frac{x}{2}\right)$:

$$= \frac{2}{5\left(\tan\left(\frac{x}{2}\right)-1\right)} - \frac{2\arctan\left(\frac{4\tan\left(\frac{x}{2}\right)+1}{\sqrt{15}}\right)}{5\sqrt{15}}$$

The problem is solved:

$$\int \frac{1}{(\sin(x)-1)(\sin(x)+4)}\,dx$$

$$= \frac{2}{5\left(\tan\left(\frac{x}{2}\right)-1\right)} - \frac{2\arctan\left(\frac{4\tan\left(\frac{x}{2}\right)+1}{\sqrt{15}}\right)}{5\sqrt{15}} + C$$