

$$\begin{aligned}
 & \int \int_S \left( \frac{2x}{\pi} + \sin(y^2) \right) x + \left( e^z - \frac{y}{\pi} \right) y + \left( \frac{2z}{\pi} + \sin^2 y \right) z \, d\sigma \\
 &= \int \int_S \left( \frac{2x}{\pi} + \sin(y^2) \right) \hat{i} + \left( e^z - \frac{y}{\pi} \right) \hat{j} + \left( \frac{2z}{\pi} + \sin^2 y \right) \hat{k} \cdot \hat{n} \, dS \\
 & \text{since region is closed, use Divergence Theorem}
 \end{aligned}$$

$$\Rightarrow \iiint_V \nabla \cdot \mathbf{F} \, dV = \iiint_V \left( \frac{2}{\pi} - \frac{1}{\pi} + \frac{2}{\pi} \right) dV = \frac{3}{\pi} \times \frac{4}{3} \times \pi \times (1)^3 = \boxed{4}$$