Q Prove that $\frac{x^2}{a^2+h} + \frac{y^2}{b^2+h} = 1$ is self orthogonal.

Sol $\frac{2x}{a^2+h} + \frac{2yy'}{b^2+h} = 0$

$xb^2 + xh + a^2yy' + hyy' = 0$

$(x+yy')h = -(xb^2+a^2yy')$

$h = -\left(\frac{xb^2+a^2yy'}{x+yy'}\right)$

$a^2+h = a^2 - \left(\frac{xb^2+a^2yy'}{x+yy'}\right) = \frac{a^2x + a^2yy' - xb^2 - a^2yy'}{x+yy'}$

$= \frac{(a^2-b^2)x}{x+yy'}$

$b^2+h = b^2 - \left(\frac{xb^2+a^2yy'}{x+yy'}\right) = \frac{b^2x + b^2yy' - xb^2 - a^2yy'}{x+yy'}$

$= \frac{(b^2-a^2)yy'}{x+yy'}$

$$\frac{x^2(x+yy')}{(a^2-b^2)x} + \frac{y^2(x+yy')}{(b^2-a^2)yy'} = 1$$

$$(x+yy')\left[\frac{xy'-y}{(a^2-b^2)y'}\right] = 1$$

$$(x+yy')(xy'-y) = (a^2-b^2)y' \quad \text{--- (2)}$$

$$y' \to -\frac{1}{y'}$$

$$\left(x-\frac{y}{y'}\right)\left(-\frac{x}{y'}-y\right) = -\frac{(a^2-b^2)}{y'}$$

$$-\frac{(xy'+y)(x+yy')}{y'} = -\frac{(a^2-b^2)}{y'} \quad \text{--- (3)}$$

(2) & (3) are same.

Thus it is ~~not~~ self orthogonal.