

Since ϕ is solⁿ of Laplace $\Rightarrow \boxed{\nabla^2 \phi = 0}$

① check solenoidal

$$\nabla(\nabla\phi + \eta) = \nabla^2\phi + \nabla\eta = 0 + 3 = 3 \neq 0$$

$\Rightarrow$ not solenoidal

② check irrotational

$$\nabla\phi = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}$$

$$\nabla \times (\nabla\phi + \eta) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial\phi}{\partial x} + 2 & \frac{\partial\phi}{\partial y} + y & \frac{\partial\phi}{\partial z} + z \end{vmatrix} = \hat{i}(0) - \hat{j}(0) + \hat{k}(0-0) = 0$$

$\Rightarrow$ irrotational ✓