$\sum_{n=1}^{\infty} (-1)^n n^p \log n$ [replace k by n]

① to check absolute convergence

$\left|\sum_{n=1}^{\infty} (-1)^n n^p \log n\right| = \sum_{n=1}^{\infty} n^p \log n$ → use cauchy condensation test

$\Rightarrow$ ($\sum a_n$ & $\sum 2^n a_{2^n}$ behave alike)

$\Rightarrow \sum 2^n 2^{np} \log 2^n \Rightarrow \sum n 2^{n(p+1)} \log 2$

now use cauchy root test

$\lim_{n\to\infty} (a_n)^{\frac{1}{n}} = \lim_{n\to\infty} (n 2^{n(p+1)} \log 2)^{\frac{1}{n}} = \lim_{n\to\infty} n^{\frac{1}{n}} \times 2^{p+1} (\log 2)^{\frac{1}{n}}$

$= 1 \times 2^{p+1} \times 1 = 2^{p+1} < 1 = 2^0$

$\Rightarrow p+1 < 0 \Rightarrow \boxed{p < -1}$

option (b)

option (a) take $p = 0$ then $\sum (-1)^k \log k$ → diverges by Leibnitz test ✗

option (c) ✗ because absolute convergent at $p = -2$, proved above.

option (d) True because ~~$\sum \log k$~~ $\sum (-1)^k \frac{\log k}{k^p}$ for $p > 1$ is convergent by Leibnitz.