

$$\phi \text{ is harmonic f.n.} \Rightarrow \boxed{\nabla^2 \phi = 0}$$

$$(a) \quad \nabla \cdot (\nabla \phi) = \nabla^2 \phi = 0 \Rightarrow \underline{\nabla \phi \text{ is solenoidal}}$$

$$(b) \quad \nabla \times (\nabla \phi) = 0 \text{ (property)} \Rightarrow \underline{\nabla \phi \text{ is irrotational}}$$

$$(c) \quad \nabla (\nabla \phi \times \nabla \psi) = (\nabla \psi) \cdot (\nabla \times \nabla \phi) - (\nabla \phi) \cdot (\nabla \times \nabla \psi) = 0$$

$\Downarrow \qquad \qquad \qquad \Downarrow$

$$\Rightarrow \underline{(\nabla \phi \times \nabla \psi) \text{ is solenoidal}}$$

option (d) ✓

Note $\rightarrow$ ① $\nabla \cdot (\vec{f} \times \vec{g}) = \vec{g} \cdot (\nabla \times \vec{f}) - \vec{f} \cdot (\nabla \times \vec{g})$ [where $\vec{f}$ & $\vec{g}$ are vector functions]

② $\nabla \times (\nabla \phi) = 0$ [ϕ is scalar function]