$$f(x) = \frac{\sin \pi x}{\pi \sin x}$$

$$f'(x) = \frac{\pi^2 \sin x (\cos \pi x) - \pi \sin \pi x \times \cos x}{(\pi \sin x)^2}$$

Given $f'(x_0) = 0$

$$\Rightarrow \pi^2 \sin x_0 \cdot \cos \pi x_0 = \pi \sin(\pi x_0) \cdot \cos x_0$$

$$\Rightarrow \boxed{\frac{\sin(\pi x_0)}{\pi \sin x_0} = \frac{\cos(\pi x_0)}{\cos(x_0)}} \quad \text{— (1)}$$

Consider $(f(x_0))^2 (1 + \pi^2 \sin^2 x_0 - \sin^2 x_0)$

$$\Rightarrow \left(\frac{\sin \pi x_0}{\pi \sin x_0}\right)^2 (\cos^2 x_0 + \pi^2 \sin^2 x_0)$$

$$\Rightarrow \frac{\cos^2 \pi x_0}{\cos^2 x_0} \times \cos^2 x_0 \left(1 + \pi^2 \frac{\sin^2 x_0}{\cos^2 x_0}\right) \quad \text{(from (1))}$$

$$\Rightarrow \cos^2 \pi x_0 \left[1 + \pi^2 \frac{\sin^2 \pi x_0}{\pi^2 \cos^2 \pi x_0}\right] \quad \text{(from (1))}$$

$$\Rightarrow \cos^2 \pi x_0 + \sin^2 \pi x_0 = 1 \text{ Ans}$$