$$\boxed{a^3+b^3=(a+b)(a^2-ab+b^2)}$$

$$f(x)=\frac{(1+x^{1/3})^3+(1-x^{1/3})^3}{8(1+x)}$$

$$=\frac{2\left(1+x^{2/3}+\cancel{2x^{1/3}}+\cancel{1}+x^{2/3}-\cancel{2x^{1/3}}-\cancel{1}+x^{2/3}\right)}{8(1+x)}$$

$$=\frac{(1+3x^{2/3})}{4(1+x)}$$

$$f'(x)=\frac{2x^{-1/3}(1+x)-(1+3x^{2/3})}{4(1+x)^2}$$

$$=\frac{2+2x-x^{1/3}-3x}{4x^{1/3}(1+x)^2}=\frac{2-x^{1/3}-x}{4x^{1/3}(1+x)^2}\geq 0 \quad \forall x\in[0,1]$$

$\Rightarrow$ f is increasing

$$\max f(x)=f(1)=\frac{2^3-0}{8\times 2}=\boxed{\frac{1}{2}}$$

$$\min f(x)=f(0)=\frac{1+1}{8}=\boxed{\frac{1}{4}}$$

$$\max f(x)-\min f(x)=\frac{1}{2}-\frac{1}{4}=\boxed{\frac{1}{4}}$$ Ans