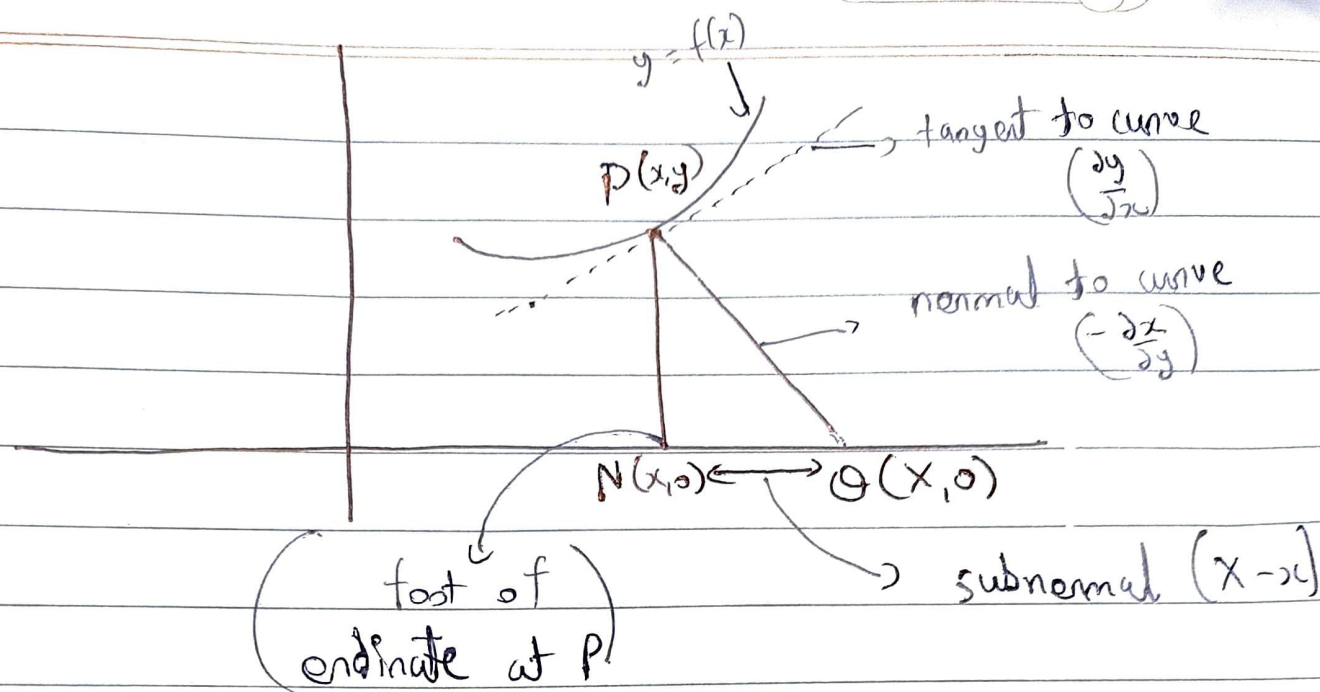




Equations of normal PQ

$$(0 - y) = -\frac{dx}{dy} (X - x)$$

$$\Rightarrow \boxed{X = x + y \frac{dy}{dx}}$$

Now, Subnormal = NQ = $X - x$ = $y \frac{dy}{dx} = \frac{x(1+y^2)}{(1+x^2)}$ (given)

$$\Rightarrow x + y \frac{dy}{dx} - x = \frac{x(1+y^2)}{(1+x^2)}$$

$$\Rightarrow \frac{y}{1+y^2} dy = \frac{x}{1+x^2} dx$$

$$\Rightarrow \int \frac{2y}{1+y^2} dy = \int \frac{2x}{1+x^2} dx$$

$$\Rightarrow \ln(1+y^2) = \ln(1+x^2) + \ln c$$

$$\Rightarrow (1+y^2) = c(1+x^2)$$

Since curve is passing through (3, 1)

$$2 = c(1+9)$$

$$2 = 10c \Rightarrow$$

$$c = \frac{1}{5}$$

Thus, equation of curve

$$= \frac{(1+y^2)}{5} = \frac{1}{5}(1+x^2)$$

$$\Rightarrow 5 + 5y^2 = 1 + x^2$$

$$\Rightarrow \boxed{x^2 - 5y^2 = 4} \quad h$$

A segment of a line normal to a tangent lying between the tangency point and the intercept of the normal with the x-axis is called the length of the normal.

A segment of the x-axis lying between the x-coordinate of the tangency point and the intercept of the normal with the axis is called the subnormal.

Therefore, the subnormal is the projection of the segment of the normal onto the x-axis.

In the figure below denoted are,

the length of the tangent $t_l = PT$, the subtangent $s_t = TX$,

the length of the normal $n_l = PN$ and the subnormal $s_n = NX$.

