$$(y + xy^2)dx + (x - yx^2)dy = 0$$

$$\frac{\partial M}{\partial y} = 1 + 2xy$$

$$\frac{\partial N}{\partial x} = 1 - 2xy$$

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 1 + 2xy - 1 + 2xy$$
$$= 4xy.$$

$$\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)\frac{1}{N} = \frac{4xy}{x(1 - xy)}$$ (can't be taken IF)

So,

$$y\,dx + xy^2dx + x\,dy - yx^2dy = 0$$

$$(ydx + xdy) + (xy^2dx - yx^2)dy = 0$$

$$x^0y^0(ydx + xdy) + xy(ydx - xdy) = 0$$

Here $a_1 = 0, b_1 = 0, m_1 = 1, n_1 = 1$

$a_2 = 1, b_2 = 1, m_2 = 1, n_2 = -1$

$$x^{Km-a-1}\,y^{Kn-b-1} = \text{IF of eq}^n$$

So for 1st part $x^{K_1 - 0 - 1}\,y^{K_1 - 0 - 1} \Rightarrow x^{K_1 - 1}\,y^{K_1 - 1}$

For 2nd part $x^{K_2 - 1 - 1}\,y^{-K_2 - 1 - 1}$

$= x^{K_2 - 2}\,y^{-2 - K_2}$

Now equating x & y (same factor)

$$k_1 - 1 = k_2 - 2 \quad \text{— (I)}$$

$$k_1 - 1 = -2 - k_2 \quad \text{— (II)}$$

$$k_1 - k_2 = -1$$

$$k_1 + k_2 = -1$$

$$2k_1 = -2 \Rightarrow k_1 = -1, \quad \therefore k_2 = -1 + 1 = 0$$

Common factor $= x^{-1-1} y^{-1-1} = x^{-2} y^{-2} = \frac{1}{x^2 y^2}$

$$\left(\frac{1}{x^2} + \frac{y}{x}\right) dx + \left(\frac{1}{xy} - 1\right) dy = 0$$

$$\frac{\partial M}{\partial y} = \frac{1}{x}, \qquad \frac{\partial N}{\partial x} = \frac{1}{x^2 y}$$

Multiply with IF $= \frac{1}{x^2 y^2}$

$$\left(\frac{1}{x^2 y} + \frac{1}{x}\right) dx + \left(\frac{1}{xy^2} - \frac{1}{y}\right) dy = 0$$

$$\frac{\partial M}{\partial y} = \frac{-1}{x^2 y^2} \Rightarrow \frac{\partial N}{\partial x} = -\frac{1}{y^2 x^2} \quad \text{(Exact)}$$

$$\therefore \int \frac{1}{x^2 y} + \frac{1}{x} dx + \int -\frac{1}{y} dy = 0$$

$$-\frac{1}{y} + \log x - \log y = C$$