

$$|\tan^{-1}x - \tan^{-1}y| = \left| \tan^{-1}\left(\frac{x-y}{1+xy}\right) \right|$$

$$\text{for } x, y \in (0, \infty), \quad \tan^{-1}x \leq x$$

$$\leq \left| \frac{x-y}{1+xy} \right| \leq |x-y| \quad [(1+xy) \text{ is +ve}]$$

Thus

$$|\tan^{-1}x - \tan^{-1}y| \leq |x-y| \quad \checkmark$$