

$$f(x,y) = -x^3y - xy^3 + 4xy$$

$$f_x = -3x^2y - y^3 + 4y \quad \wedge \quad f_y = -x^3 - 3xy^2 + 4x$$

$$f_x = 0 \quad \wedge \quad f_y = 0 \text{ gives}$$

$$y(-3x^2 - y^2 + 4) = 0$$

$$y = 0, \quad y = \pm \sqrt{4 - 3x^2}$$

$$x(-x^2 - 3y^2 + 4) = 0$$

$$x = 0, \quad x = \sqrt{4 - 3y^2}$$

$$y = \sqrt{4 - 3(4 - 3y^2)} = \sqrt{4 - 12 + 9y^2}$$

$$y = \pm 1 \Rightarrow x = \pm 1$$

Critical points are - $(1,1), (1,-1), (-1,1), (-1,-1), (0,2), (0,-2), (2,0), (-2,0)$

$$\text{Since } |x| < 3/2 \quad \wedge \quad |y| < 3/2$$

So critical $\Rightarrow (1,1), (1,-1), (-1,1), (-1,-1)$

$$f_{xy} = -3x^2 - 3y^2 + 4$$

$$\Rightarrow \frac{f_{xx} \cdot f_{yy} - (f_{xy})^2}{36x^2y^2 - (3x^2 + 3y^2 - 4)^2} = \frac{(-6xy)(-6xy) - (-3x^2 - 3y^2 + 4)^2}{36x^2y^2 - (3x^2 + 3y^2 - 4)^2}$$

① at (1,1)

$$36 - (3+3-4)^2 \Rightarrow 36 - 4 = 32 > 0$$

$$f_{xx}(1,1) = -6 < 0 \rightarrow \text{maximum}$$

② at (1,-1)

$$36 - (3+3-4)^2 \Rightarrow 32 > 0$$

$$f_{xx}(1,-1) = 6 > 0 \rightarrow \text{minimum}$$

③ at (-1,1)

$$36 - (3+3-4)^2 = 32 > 0$$

$$f_{xx}(-1,1) = 6 > 0 \rightarrow \text{minimum}$$

④ at (-1,-1)

maximum

minimum at $(-1, 1)$ and $(1, -1)$

$$\underline{f(1, -1)} = (1)^3(-1) - (1)(-1)^3 + 4(1)(-1) = 1 + 1 - 4 \Rightarrow \boxed{-2}$$

$$f(-1, 1) = -(-1)^3(1) - (-1)(1)^3 + 4(-1)(1) = 1 + 1 - 4 \Rightarrow \boxed{-2}$$

So minimum value is -2 at $(1, -1)$ and $(-1, 1)$