

$$\textcircled{a} \quad f(x) = (\ln(x))^2$$

$$\textcircled{a} \quad \lim_{x \rightarrow \infty} \frac{f(x)}{x} = \lim_{x \rightarrow \infty} \frac{(\ln x)^2}{x} \quad \left(\frac{\infty}{\infty}\right) \text{ form, use l'Hopital rule}$$

$$= \frac{2 \ln x}{1} \Rightarrow \frac{2}{x} = \boxed{0} \quad \text{exist}$$

$$\textcircled{b} \quad \lim_{x \rightarrow \infty} f'(x) = \lim_{x \rightarrow \infty} \frac{2 \ln x}{x} = \boxed{0} \neq 2 \quad x$$

$$\textcircled{c} \quad \lim_{x \rightarrow \infty} [\ln(x+1)]^2 - (\ln x)^2 =$$

$$\lim_{x \rightarrow \infty} [(\cancel{\ln 2})^2 - (\ln 1)^2] + [(\cancel{\ln 3})^2 - (\cancel{\ln 2})^2] + [(\ln 4)^2 - (\ln 3)^2] + \dots$$

$$[(\ln x+1)^2 - (\ln x)^2]$$

$$\lim_{x \rightarrow \infty} 0 + (\ln(x+1))^2 = \boxed{\infty} \quad \text{does not exist}$$

option (b) is correct