$$\int_0^{\frac{1}{2}} \frac{x \sin^{-1} x}{\sqrt{1-x^2}}\, dx$$

let $\sin^{-1} x = t \Rightarrow \frac{dt}{dx} = \frac{1}{\sqrt{1-x^2}}$

$$\Rightarrow \int_0^{\frac{1}{2}} \frac{x\,t}{\sqrt{1-x^2}} \times \sqrt{1-x^2}\, dt$$

$$\Rightarrow \int_0^{\frac{1}{2}} t \sin t\, dt$$

$$\Rightarrow \left[-t\cos t + \int 1 \cdot \cos t\, dt \right]_0^{\frac{1}{2}}$$

$$\Rightarrow \left[-t\cos t + \sin t \right]_0^{\frac{1}{2}}$$

$$\Rightarrow \left[-\sin^{-1} x \cos(\sin^{-1} x) + \sin \sin^{-1} x \right]_0^{\frac{1}{2}}$$

$$- \sin^{-1}\frac{1}{2} \cdot \cos\left(\sin^{-1}\frac{1}{2}\right) + \frac{1}{2} - [0]$$

$$\Rightarrow -\frac{\pi}{6} \times \frac{\sqrt{3}}{2} + \frac{1}{2}$$

$$\Rightarrow \boxed{-\frac{\sqrt{3}\pi - 6}{12}}$$