

$$(3x^2y^4 + 2xy) dx + (2x^3y^3 - x^2) dy = 0$$

$$M = 3x^2y^4 + 2xy, \quad N = 2x^3y^3 - x^2$$

$$\frac{\partial M}{\partial y} = 12x^2y^3 + 2x, \quad \frac{\partial N}{\partial x} = 6x^2y^3 - 2x$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

So equation
is not exact.

$$\frac{\partial m}{\partial y} \neq \frac{\partial n}{\partial x}$$

So equation is not exact.

Then we try to find Integrating factor

$$f(y) = \frac{1}{n} \left(\frac{\partial n}{\partial x} - \frac{\partial m}{\partial y} \right)$$

$$= -\frac{2}{y}$$

$$\text{I.F.} = e^{-2/y} dy = \frac{1}{y^2}$$

Multiply given equation by I.F.

$$(3x^2y^2 + \frac{2x}{y}) dx + (2x^3y - \frac{x^2}{y^2}) dy = 0$$

This equation is ~~an~~ exact

Solution.

$$\int M dx + \int (\text{terms of } x \text{ containing } y \text{ only}) dy = C$$

$$x^3y^2 + \frac{x^2}{y} + 0 = C$$