Problem:

$$\int \frac{ze^z}{(1-e^z)^2}\,dz$$

Rewrite:

$$= \int \frac{ze^z}{(e^z-1)^2}\,dz$$

Integrate by parts: $\int \mathtt{f}\mathtt{g}' = \mathtt{f}\mathtt{g} - \int \mathtt{f}'\mathtt{g}$

$$\mathtt{f} = z, \quad \mathtt{g}' = \frac{e^z}{(e^z-1)^2}$$

$\downarrow$ steps $\quad$ $\downarrow$ steps

$$\mathtt{f}' = 1, \quad \mathtt{g} = -\frac{1}{e^z-1}:$$

$$= -\frac{z}{e^z-1} - \int -\frac{1}{e^z-1}\,dz$$

Now solving:

$$\int -\frac{1}{e^z-1}\,dz$$

Apply linearity:

$$= -\int \frac{1}{e^z-1}\,dz$$

Now solving:

$$\int \frac{1}{e^z - 1}\, dz$$

Substitute $u = e^z - 1 \longrightarrow \frac{du}{dz} = e^z$ (steps) $\longrightarrow$ $dz = e^{-z}\, du$:

$$= \int \frac{1}{u\,(u+1)}\, du$$

... or choose an alternative:

Substitute e^z

This integral could be solved using partial fraction decomposition, but there is a simpler way.

Expand fraction by $\frac{1}{u^2}$:

$$= \int \frac{1}{\left(\frac{1}{u} + 1\right) u^2}\, du$$

... or choose an alternative:

Don't apply this trick

Substitute $v = \frac{1}{u} + 1 \longrightarrow \frac{dv}{du} = -\frac{1}{u^2}$ (steps) $\longrightarrow$ $du = -u^2\, dv$:

$$= -\int \frac{1}{v}\, dv$$

Now solving:

$$\int \frac{1}{v}\, dv$$

This is a standard integral:

$$= \ln(v)$$

Plug in solved integrals:

$$-\int \frac{1}{v}\,\mathrm{d}v$$

$$= -\ln(v)$$

Undo substitution $v = \frac{1}{u} + 1$:

$$= -\ln\left(\frac{1}{u} + 1\right)$$

Undo substitution $u = \mathrm{e}^z - 1$:

$$= -\ln\left(\frac{1}{\mathrm{e}^z - 1} + 1\right)$$

Plug in solved integrals:

$$-\int \frac{1}{\mathrm{e}^z - 1}\,\mathrm{d}z$$

$$= \ln\left(\frac{1}{\mathrm{e}^z - 1} + 1\right)$$

Plug in solved integrals:

$$-\frac{z}{\mathrm{e}^z - 1} - \int -\frac{1}{\mathrm{e}^z - 1}\,\mathrm{d}z$$

$$= -\ln\left(\frac{1}{\mathrm{e}^z - 1} + 1\right) - \frac{z}{\mathrm{e}^z - 1}$$

The problem is solved. Apply the absolute value function to arguments of logarithm functions in order to extend the antiderivative's domain:

$$\int \frac{z\mathrm{e}^z}{(1 - \mathrm{e}^z)^2}\,\mathrm{d}z$$

$$= -\ln\left(\left|\frac{1}{\mathrm{e}^z - 1} + 1\right|\right) - \frac{z}{\mathrm{e}^z - 1} + C$$

ANTIDERIVATIVE COMPUTED BY MAXIMA:

$\int f(z)\, dz = F(z) =$

$$\ln(|e^z - 1|) - \frac{ze^z}{e^z - 1} + C$$

Simplify