$$x + y + z = 3$$
$$x + 3y + 2z = 6$$
$$x + \alpha y + 3z = \mu$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 2 \\ 1 & \alpha & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \\ \mu \end{bmatrix}$$

$$AX = B$$

Now $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 2 \\ 1 & \alpha & 3 \end{bmatrix}$; $[A/B] = \begin{bmatrix} 1 & 1 & 1 & : & 3 \\ 1 & 3 & 2 & : & 6 \\ 1 & \alpha & 3 & : & \mu \end{bmatrix}$

$$\rho(A/B) = \begin{bmatrix} 1 & 1 & 1 & : & 3 \\ 0 & 2 & 1 & : & 3 \\ 0 & \alpha-1 & 2 & : & \mu-3 \end{bmatrix}$$

① if $\alpha = 5$ then

$$\rho(A/B) = \begin{bmatrix} 1 & 1 & 1 & : & 3 \\ 0 & 2 & 1 & : & 3 \\ 0 & 4 & 2 & : & \mu-3 \end{bmatrix} \Rightarrow R_3 \to R_3 - 2R_2$$

$$\begin{bmatrix} 1 & 1 & 1 & : & 3 \\ 0 & 2 & 1 & : & 3 \\ 0 & 0 & 0 & : & \mu-9 \end{bmatrix} \quad \text{---(1)}$$

A

Now if $\mu \neq 9$ then

$\rho(A/B) = 3$ & $\rho(A) = 2$

$\rho(A) = \rho(A|B) = 3$ $\Rightarrow$ unique solution

$\rho(A) = \rho(A|B) < 3$ $\Rightarrow$ infinitely many solution

$\rho(A) \neq \rho(A|B)$ $\Rightarrow$ solution do exist not exist

So

For option (b) $\lambda \neq 5$, $\mu = 9$

Then from equation (1)

$$\begin{bmatrix} 1 & 1 & 1 & : & 3 \\ 0 & 2 & 1 & : & 3 \\ 0 & 0 & \lambda-5 & : & \mu-9 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & : & 3 \\ 0 & 2 & 1 & : & 3 \\ 0 & 0 & \lambda-5 & : & 0 \end{bmatrix}$$

$\rho(A|B) = 3$ & $\rho(A) = 3$ $\quad (\because \lambda \neq 5)$

$\Rightarrow$ unique solution in this case ($\rho(A) = \rho(A|B) = 3$

(c) if $\lambda = 5$; $\mu \neq 9$

$\rho(A) = 2$; $\rho(A|B) = 3$

$\rho(A) \neq \rho(A|B)$ $\Rightarrow$ Solution doesn't exist

Option D is correct