Given equation of line AB is

$$\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda \text{ [say]}$$

or $$\frac{x}{2} = \lambda, \frac{y-2}{3} = \lambda$$

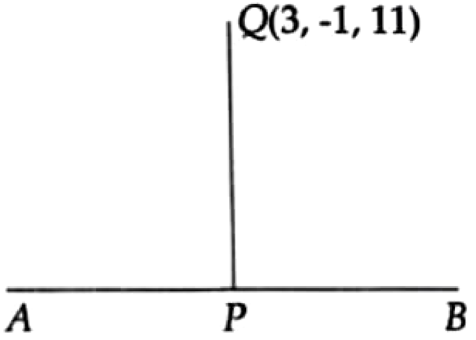
and $$\frac{z-3}{4} = \lambda$$

or $$x = 2\lambda, y = 3\lambda + 2$$

and $$z = 4\lambda + 3$$

$\therefore$ Any point $P$ on the given line

$$= (2\lambda, 3\lambda + 2, 4\lambda + 3)$$




Let $P$ be the foot of perpendicular drawn from point $Q(3, -1, 11)$ on line $AB$. Now, DR's of line

$$QP = (2\lambda - 3, 3\lambda + 2 + 1, 4\lambda + 3 - 11)$$
$$= (2\lambda - 3, 3\lambda + 3, 4\lambda - 8)$$

Here, $a_1 = 2\lambda - 3, b_1 = 3\lambda + 3, c_1 = 4\lambda - 8,$

and $a_2 = 2, b_2 = 3, c_2 = 4$

Since, $QP \perp AB$

$\therefore$ We have, $$a_1a_2 + b_1b_2 + c_1c_2 = 0 \qquad \text{...(i)}$$

or $2(2\lambda - 3) + 3(3\lambda + 3) + 4(4\lambda - 8) = 0$

or $4\lambda - 6 + 9\lambda + 9 + 16\lambda - 32 = 0$

or $29\lambda - 29 = 0$ or $29\lambda = 29$ or $\lambda = 1$ 1

$\therefore$ Foot of perpendicular

$$P = (2, 3 + 2, 4 + 3)$$
$$= (2, 5, 7)$$

Now, equation of perpendicular $QP$, where $Q$ (3, –1, 11) and $P$ (2, 5, 7), is

$$\frac{x-3}{2-3} = \frac{y+1}{5+1} = \frac{z-11}{7-11}$$

... points form of equation of line.]

Since, $QP \perp AB$

$\therefore$ We have, $a_1a_2 + b_1b_2 + c_1c_2 = 0$ ...(i)

or $2(2\lambda - 3) + 3(3\lambda + 3) + 4(4\lambda - 8) = 0$

or $4\lambda - 6 + 9\lambda + 9 + 16\lambda - 32 = 0$

or $29\lambda - 29 = 0$ or $29\lambda = 29$ or $\lambda = 1$ 1

$\therefore$ Foot of perpendicular

$$P = (2, 3 + 2, 4 + 3)$$
$$= (2, 5, 7)$$

Now, equation of perpendicular $QP$, where $Q(3, -1, 11)$ and $P(2, 5, 7)$, is

$$\frac{x-3}{2-3} = \frac{y+1}{5+1} = \frac{z-11}{7-11}$$

$$\left[\begin{array}{l}\text{using two points form of equation of line,} \\ i.e.\ \dfrac{x-x_1}{x_2-x_1} = \dfrac{y-y_1}{y_2-y_1} = \dfrac{z-z_1}{z_2-z_1}\end{array}\right]$$

or $$\frac{x-3}{-1} = \frac{y+1}{6} = \frac{z-11}{-4}$$

Now, length of perpendicular $QP$ = distance between points $Q(3, -1, 11)$ and $P(2, 5, 7)$

$$= \sqrt{(2-3)^2 + (5+1)^2 + (7-11)^2}$$

$$\left[\begin{array}{l}\because \text{ distance} \\ = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}\end{array}\right]$$

$$= \sqrt{1 + 36 + 16} = \sqrt{53}$$

Hence, length of perpendicular is $\sqrt{53}$.