

~~Let~~ $R: xy=1, xy=3, \frac{y}{x}=3, \frac{y}{x}=5$

Let $xy=u$ & $\frac{y}{x}=v$

$\Rightarrow \boxed{1 \leq u \leq 3 \quad \text{and} \quad 3 \leq v \leq 5}$

$\frac{1}{|J|} = \begin{vmatrix} y & x \\ -\frac{y}{x^2} & \frac{1}{x} \end{vmatrix} = \frac{2y}{x} \Rightarrow |J| = \frac{x}{2y} = \boxed{\frac{1}{2v}}$

$\iint_R xy \, dx \, dy = \int_{u=1}^3 \int_{v=3}^5 u \times |J| \, du \, dv$

$\Rightarrow \frac{1}{2} \int_{u=1}^3 \int_{v=3}^5 \frac{u}{v} \, du \, dv$

$\Rightarrow \frac{1}{2} \int_{u=1}^3 u \left[\log v \right]_3^5 \, du \Rightarrow \boxed{2 \log \frac{5}{3}}$