

$$\frac{1 \cdot \sqrt{n^2+1^2}}{n^3} + \frac{2 \cdot \sqrt{n^2+2^2}}{n^3} + \dots + \frac{n \cdot \sqrt{n^2+n^2}}{n^3}$$

$$\lim_{h \rightarrow \infty} \sum_{r=1}^n r \cdot \frac{\sqrt{n^2+r^2}}{n^3}$$

$$= \lim_{h \rightarrow \infty} \sum_{r=1}^n \left(\frac{r}{n} \right) \cdot \frac{\sqrt{1+(r/n)^2}}{n^2}$$

$$= \lim_{h \rightarrow \infty} \sum_{r=1}^n \left(\frac{r}{n} \right) \cdot \left(\frac{1}{n} \right) \sqrt{1+(r/n)^2}$$

$$\frac{1}{n} \rightarrow dx$$

$$\frac{r}{n} \rightarrow x$$

$$= \int_0^1 x \sqrt{1+x^2} dx$$

$$\text{put } 1+x^2 = t^2$$

$$2x dx = 2t dt$$

$$x dx = t dt$$

limit

$$x=0 \quad ; \quad t=1$$

$$x=1 \quad ; \quad t=\sqrt{2}$$

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$$= \int_1^{\sqrt{2}} x^2 dx = \left(\frac{x^3}{3} \right)_1^{\sqrt{2}}$$

$$= \frac{1}{3} (\sqrt{2})^3 - \frac{(1)^3}{3}$$

$$= \frac{2\sqrt{2}}{3} - \frac{1}{3}$$

$$= \frac{(2\sqrt{2} - 1)}{3}$$