

$$\lim_{N \rightarrow \infty} \left(\frac{\sum_{i=1}^N \frac{\log i}{i}}{(\log N)^2} \right)$$

$$\frac{\frac{\log 1}{1} + \frac{\log 2}{2} + \frac{\log 3}{3} + \dots + \frac{\log N}{N}}{(\log N)^2} < \frac{\sum_{i=1}^N \frac{\log i}{i}}{(\log N)^2} \leq \left(\frac{\log 1}{(\log N)^2} + \frac{\log 2}{(\log N)^2} + \dots + \frac{\log N}{(\log N)^2} \right)$$

$$f(N) \leq f(N) \leq h(N)$$

Now

$$\lim_{N \rightarrow \infty} g(N) = \lim_{N \rightarrow \infty} \left(\right)$$

Now

$$g(N) = \frac{\frac{\log 1}{1} + \frac{\log 2}{2} + \frac{\log 3}{3} + \dots + \frac{\log N}{N}}{(\log N)^2}$$

$$= \frac{0 + 0 + 0 + \dots + 0}{(\log N)^2}$$

$$= 0$$

$$g(N) = 0$$

$$\lim_{N \rightarrow \infty} g(N) = 0$$

Now

$$h(N) = \frac{\frac{\log 1}{1}}{(\log N)^2} + \frac{\frac{\log 2}{2}}{(\log N)^2} + \frac{\frac{\log 3}{3}}{(\log N)^2} + \dots + \frac{\frac{\log N}{N}}{(\log N)^2}$$

$$\lim_{N \rightarrow \infty} h(N) = \lim_{N \rightarrow \infty} \left(\frac{\log 1}{(\log N)^2} + \frac{\log^2}{(\log N)^2} + \dots + \frac{1}{(\log N)^2} \right)$$

$$= 0 + 0 + \dots + 0$$

$$\lim_{N \rightarrow \infty} h(N) = 0$$

So

$$\lim_{N \rightarrow \infty} g(N) \leq \lim_{N \rightarrow \infty} f(N) \leq \lim_{N \rightarrow \infty} h(N)$$

$$0 \leq \lim_{N \rightarrow \infty} f(N) \leq 0$$

$$\Rightarrow \lim_{N \rightarrow \infty} f(N) = 0$$

$$\Rightarrow \lim_{N \rightarrow \infty} f(N) = 0$$